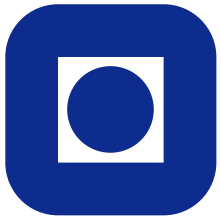

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A Short Course on Extreme Value Statistics in Applications

Arvid Naess
Department of Mathematical Sciences

Resources

Presentations can be downloaded from:

https://folk.ntnu.no/arvidn/ENBIS 2022

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My new book, which is not quite ready yet, but you are welcome to comment on it:

https://folk.ntnu.no/arvidn/BOOK_EXTREMES

Introduction

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The approach to extreme value statistics very often adopted in engineering applications has been based on the assumption that exceedances above high thresholds can be assumed independent.

It is not always clear to what extent that is an acceptable approximation. A method for extreme value statistics that is capable of accounting for statistical dependence in the data series is therefore highly desirable.

A number of approximate methods for dealing with dependence have been proposed over the years, but none of these methods were satisfactory.

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Introduction

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The final step is then to find a suitable method for making predictions based on the empirical extreme value distribution.

Introduction

Standard methods for estimating extreme values from limited sets of observed data are commonly based on assuming either that the distribution of epochal extreme values converges to a Gumbel (type-I) extreme value distribution, or by adopting a so-called generalized extreme value (GEV) distribution, which would include all three types of asymptotic extreme value distributions.

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Another commonly used approach is adopting a peaks-over-threshold (POT) method, assuming that the exceedances above high thresholds follow a generalized Pareto (GP) distribution.

Introduction

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The main problem with this is that the assumption about asymptotic behaviour cannot be fully verified for the measured data used in the extreme value analysis, and in reality it has to be adopted basically by faith or convenience.

Introduction

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The result is an asymptotically inconsistent distribution, which may have a significant impact on the estimation of long return period extreme value levels.

Classical Extreme Value Theory

The classical extreme value theory starts by looking at a sequence of independent and identically distributed (iid) random variables $X_1, X_2, \ldots$ with common distribution function $F_X(x)$. The extreme value of a finite number $X_1, \ldots, X_n$ is then $M_n = \max\{X_1, \ldots, X_n\}$.

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The distribution of M_n can be easily derived as

$$F_{M_n}(x) = \text{Prob}(M_n \leq x) = \text{Prob}(X_1 \leq x, \ldots, X_n \leq x) = \big(F_X(x)\big)^n,$$

which is not useful in practice!

Classical Extreme Value Theory

Instead of studying M_n, one introduces a renormalized version of M_n:

$$M_n^* = \frac{M_n - b_n}{a_n}$$

for suitable sequences of constants $a_n > 0$ and b_n that are chosen to stabilize the location and scale of M_n^* as $n \to \infty$.

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It is then proven that there are, in fact, only three types of limiting distributions for this renormalized M_n^*. This is the famous Extremal Types Theorem.

The Extremal Types Theorem

If there exist sequences of constants $a_n > 0$ and b_n such that

$$\text{Prob}\left(\frac{M_n - b_n}{a_n} \leq x\right) \to G(x)\,, \quad n \to \infty,$$

where $G(x)$ is a nondegenerate distribution function, then $G(x)$ belongs to one of the following three families:

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I $$G(x) = \exp\left\{-\exp\left[-\left(\frac{x-b}{a}\right)\right]\right\}\,, \quad -\infty < x < \infty\,; \qquad (1)$$

The Extremal Types Theorem

II $$G(x) = \begin{cases} 0 & , \quad x \leq b\,, \\ \exp\left\{-\left(\frac{x-b}{a}\right)^{-c}\right\} & , \quad x > b\,; \end{cases}$$

The Extremal Types Theorem

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$$\text{III} \qquad G(x) = \begin{cases} \exp\left\{-\left(\frac{b-x}{a}\right)^{c}\right\} & , \quad x < b\,, \\ 1 & , \quad x \geq b\,; \end{cases}$$

for parameters $a > 0$, b and $c > 0$.

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The Extremal Types Theorem

The ETT expressed by the Generalized Extreme Value (GEV) distribution:

$$G(x;\lambda,\delta,\kappa) = \exp\left\{ -\left[1+\kappa\left(\frac{x-\lambda}{\delta}\right)\right]_+^{-1/\kappa}\right\}$$

where $\kappa \neq 0$ and $[z]_+ = \max(0, z)$.

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When $\kappa = 0$,

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$\kappa < 0$: Reverse Weibull, $\kappa = 0$: Gumbel, $\kappa > 0$: Fréchet.

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Model Validation

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Assume that the sample of block maxima has been ordered by increasing value: $z_{(1)} \leq z_{(2)} \leq \ldots \leq z_{(k)}$.

The empirical distribution function, $\tilde{G}$ say, evaluated at $z_{(i)}$ is given by,

$$\tilde{G}(z_{(i)}) = i/(k+1).$$

Model Validation

The proposed GEV model is obtained by substituting the parameter estimates:

$$\hat{G}(z_{(i)}) = \exp\left\{-\left[1+\hat{\gamma}\left(\frac{z_{(i)}-\hat{\mu}}{\hat{\sigma}}\right)\right]^{-1/\hat{\gamma}}\right\},$$

provided $\hat{\gamma} \neq 0$. If $\hat{\gamma} = 0$, the plot is constructed using the Gumbel distribution.

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provided $\hat{\gamma} \neq 0$. If $\hat{\gamma} = 0$, the plot is constructed using the Gumbel distribution.

If the GEV model is a good approximation, then the PP plot consisting of the points

$$\left(\hat{G}(z_{(i)}), \tilde{G}(z_{(i)})\right) \quad i = 1, \ldots, k,$$

should follow approximately the unit diagonal.

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Model Validation

For the case of extreme value distributions, a quantile or QQ plot is usually considered to be more informative than a PP plot because it shows more clearly the agreement at high values of the observed data, which is of primary concern when fitting extreme value models.

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For $\hat{\gamma} \neq 0$, the QQ plot is traced out by the point graph,

$$\left(\hat{G}^{-1}(i/(k+1)), z_{(i)}\right), \quad i = 1, \ldots, k,$$

where

$$\hat{G}^{-1}(i/(k+1)) = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\gamma}}\left[1 - \left\{-\log\left(i/(k+1)\right)\right\}^{-\hat{\gamma}}\right].$$

This graph should also approximately follow a straight line.

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Confidence Intervals by Bootstrapping

The bootstrapping method is based on resampling from a distribution determined by the available sample of data, either parametric or nonparametric.

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Confidence Intervals by Bootstrapping

The bootstrapping method is based on resampling from a distribution determined by the available sample of data, either parametric or nonparametric.

Assume that $\mathbf{z} = (z_1, z_2, \ldots, z_n)$ is a sample or vector consisting of n independent observations of a random variable Z.

Parametric: Z has a specified distribution function $F_Z(z;\theta) = \text{Prob}(Z \leq z)$, where θ denotes a vector of unknown parameters, which determine the distribution. These parameters are then estimated from the observed data $\mathbf{z}$, giving $\hat{\theta}$, and $F_Z(z;\hat{\theta})$ is adopted as the distribution of Z.

Confidence Intervals by Bootstrapping

Nonparametric: A purely empirical distribution function is established for Z on the basis of the observed data by allocating a probability of $1/n$ to each of the observed data points.

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The goal is to estimate some statistical quantity V, e.g. a high quantile like $100(1-\alpha)\%$ ($0 < \alpha << 1$), given by the unknown distribution.

Let $\hat{V}$ denote the estimate of V obtained from the fitted model distribution $F_Z(z;\hat{\theta})$, which is a GEV distribution.

Confidence Intervals by Bootstrapping

Resampling: Let Z^* denote the random variable with distribution function $F_Z(z;\hat{\theta})$. ℓ bootstrap samples $\mathbf{z}_j^*$, $j = 1,\ldots,\ell$, with n independent observations of Z^* in each sample are now generated. Each sample $\mathbf{z}_j^*$ is used to fit a new GEV model from which an estimate V_j^* of V is obtained.

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A simple estimator for confidence intervals on $\hat{V}$ is derived by calculating the sample standard deviation s_V^*:

$$s_V^* = \sqrt{\frac{1}{\ell - 1} \sum_{j=1}^{\ell} (V_j^* - \bar{V}^*)^2},$$

where $\bar{V}^* = (1/\ell) \sum_{j=1}^{\ell} V_j^*$.

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Confidence Intervals by Bootstrapping

An approximate confidence interval at level $1 - q$ is then obtained as,

$$(\hat{V} - w_{q/2}\, s_V^* ,\ \hat{V} + w_{q/2}\, s_V^*), \tag{2}$$

where $w_{q/2}$ denotes the $100(1 - q/2)\%$ standard normal fractile.

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To avoid making the assumption that the bootstrap estimates are normally distributed, the true distribution may be approximated by generating a large number of bootstrap samples, usually several thousand are needed, especially for small values of q.

Confidence Intervals by Bootstrapping

If ℓ samples are generated, the V_j^* are rearranged in increasing order. A $100(1-q)\%$ confidence interval is then,

$$(V_L^*, V_M^*),$$

where $L = [q\ell/2]$ and $M = [(1-q/2)\ell]$ ($[a]$ means the integer part of a).

The Peaks-Over-Threshold Method

If the distribution of maxima of some sequence of independent and identically distributed random variables $X_1, X_2, \ldots$ converges asymptotically to a generalized extreme value distribution with parameters λ, δ and κ, as expressed in the GEV, then the exceedances given by $Y = X - u$ of some high threshold u, conditional on $X > u$, are approximately distributed as the Generalized Pareto (GP) distribution:

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$$H(y; \lambda, \delta, \kappa) = H(y) = 1 - \left(1 + \kappa \frac{y}{\tilde{\delta}}\right)^{-1/\kappa}$$

where $\kappa \neq 0$, $\{y : y > 0, (1 + \kappa y/\tilde{\delta}) > 0\}$, and $\tilde{\delta} = \delta + \kappa(u - \lambda)$ with κ equal to that of the corresponding GEV.

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The Peaks-Over-Threshold Method

When $\kappa = 0$, the GP distribution becomes an exponential distribution:

$$H(y; \lambda, \delta, 0) = 1 - \exp\left(-\frac{y}{\tilde{\delta}}\right)$$

for $y > 0$.

The ACER Method

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Cascade of Approximations

Let $0 \leq t_1 < \ldots < t_N \leq T$ denote the points in time for the observed data values of $X(t)$, and let $X_k = X(t_k)$, $k = 1, ..., N$.

Cascade of Approximations

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$$
\begin{aligned}
P(\eta) &= \text{Prob}\{X_1 \leq \eta, \ldots, X_N \leq \eta\} \\
&= \text{Prob}\{X_N \leq \eta | X_1 \leq \eta, \ldots, X_{N-1} \leq \eta\} \text{Prob}\{X_1 \leq \eta, \ldots, X_{N-1} \leq \eta\} \\
&= \prod_{j=2}^{N} \text{Prob}\{X_j \leq \eta | X_1 \leq \eta, \ldots, X_{j-1}\} \cdot P(X_1 \leq \eta)
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\end{aligned}
$$

For independent values with $\alpha_{1j}(\eta) = \text{Prob}\{X_j > \eta\}$,

$$
P(\eta) \approx \prod_{j=1}^{N} P(X_j \leq \eta) = \prod_{j=1}^{N} \left(1 - \alpha_{1j}(\eta)\right) \approx P_1(\eta) = \exp\left(-\sum_{j=1}^{N} \alpha_{1j}(\eta)\right)
$$

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Cascade of Approximations

Conditioning on one previous value:

$$\text{Prob}\{X_j \leq \eta | X_1 \leq \eta, ..., X_{j-1} \leq \eta\} \approx \text{Prob}\{X_j \leq \eta | X_{j-1} \leq \eta\}\,.$$

This leads to the approximation:

$$P(\eta) \approx P_2(\eta) = \exp\Big(-\sum_{j=2}^{N} \alpha_{2j}(\eta) - \alpha_{11}(\eta)\Big)\,,$$

where $\alpha_{2j}(\eta) = \text{Prob}\{X_j > \eta \,|\, X_{j-1} \leq \eta\}$.

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Cascade of Approximations

Two more approximations:

$$P(\eta) \approx P_3(\eta) = \exp\Big(-\sum_{j=3}^{N} \alpha_{3j}(\eta) - \alpha_{22}(\eta) - \alpha_{11}(\eta)\Big),$$

where $\alpha_{3j}(\eta) = \text{Prob}\{X_j > \eta \,|\, X_{j-1} \leq \eta, X_{j-2} \leq \eta\}$.

$$P(\eta) \approx P_4(\eta) = \exp\Big(-\sum_{j=4}^{N} \alpha_{4j}(\eta) - \alpha_{33}(\eta) - \alpha_{22}(\eta) - \alpha_{11}(\eta)\Big),$$

where $\alpha_{4j}(\eta) = \text{Prob}\{X_j > \eta \,|\, X_{j-1} \leq \eta, X_{j-2} \leq \eta, X_{j-3} \leq \eta\}$.

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$$P(\eta) \approx P_4(\eta) = \exp\Big(-\sum_{j=4}^{N} \alpha_{4j}(\eta) - \alpha_{33}(\eta) - \alpha_{22}(\eta) - \alpha_{11}(\eta)\Big),$$

where $\alpha_{4j}(\eta) = \text{Prob}\{X_j > \eta \,|\, X_{j-1} \leq \eta, X_{j-2} \leq \eta, X_{j-3} \leq \eta\}$.
For most practical applications $N >> k$, so that

$$P_k(\eta) \approx \exp\Big(-\sum_{j=k}^{N} \alpha_{kj}(\eta)\Big)\,, \quad k = 1, 2, \ldots.$$

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Empirical Estimation of ACER

We introduce the concept of average conditional exceedance rates (ACER) as follows,

$$\varepsilon_k(\eta) = \frac{1}{N-k+1} \sum_{j=k}^{N} \alpha_{kj}(\eta)\,,\ k = 1, 2, \ldots$$

Empirical Estimation of ACER

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$$\varepsilon_k(\eta) = \frac{1}{N-k+1} \sum_{j=k}^{N} \alpha_{kj}(\eta)\,,\ k = 1, 2, \ldots$$

The following random functions are defined,

$$A_{kj}(\eta) = \mathbf{1}\{X_j > \eta, X_{j-1} \leq \eta, \ldots, X_{j-k+1} \leq \eta\}\,, j = k, \ldots, N, k = 2, 3, \ldots$$

and

$$B_{kj}(\eta) = \mathbf{1}\{X_{j-1} \leq \eta, \ldots, X_{j-k+1} \leq \eta\}\,,\ j = k, \ldots, N,\ k = 2, \ldots\,,$$

where $\mathbf{1}\{\mathcal{A}\}$ denotes the indicator function of some event $\mathcal{A}$.

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Empirical Estimation of ACER

Then

$$\alpha_{kj}(\eta) = \frac{\mathrm{E}[A_{kj}(\eta)]}{\mathrm{E}[B_{kj}(\eta)]}\,,\ j = k, \ldots, N,\ k = 2, \ldots,$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.

Empirical Estimation of ACER

Then

$$\alpha_{kj}(\eta) = \frac{\mathrm{E}[A_{kj}(\eta)]}{\mathrm{E}[B_{kj}(\eta)]}\,,\; j = k, \ldots, N,\; k = 2, \ldots,$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator.
Assuming an ergodic process, then obviously
$\varepsilon_k(\eta) = \alpha_{kk}(\eta) = \ldots = \alpha_{kN}(\eta)$, and it may be assumed that,

$$\varepsilon_k(\eta) = \lim_{N\to\infty} \frac{\sum_{j=k}^{N} a_{kj}(\eta)}{\sum_{j=k}^{N} b_{kj}(\eta)}\,,$$

where $a_{kj}(\eta)$ and $b_{kj}(\eta)$ are the realized values of $A_{kj}(\eta)$ and $B_{kj}(\eta)$, respectively, for the observed time series.

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Empirical Estimation of ACER

Clearly, $\lim_{\eta\to\infty} \sum_{j=k}^{N} \mathrm{E}[B_{kj}(\eta)] = N - k + 1 \approx N$. Hence, $\lim_{\eta\to\infty} \tilde{\varepsilon}_k(\eta)/\varepsilon_k(\eta) = 1$, where the modified ACER function $\tilde{\varepsilon}_k(\eta)$ is

$$\tilde{\varepsilon}_k(\eta) = \frac{\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)]}{N - k + 1}\,.$$

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$$\tilde{\varepsilon}_k(\eta) = \frac{\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)]}{N - k + 1}.$$

This is very convenient for nonstationary time series.

Empirical Estimation of ACER

It is of interest to note what events are actually counted for the calculation of $\tilde{\varepsilon}_2(\eta)$.

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$\tilde{\varepsilon}_2(\eta)\,(N-1)$ can be interpreted as the expected number of exceedances above the level η satisfying the condition that an exceedance is counted only if it is immediately preceded by a non-exceedance.

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A reinterpretation of this is that $\hat{\varepsilon}_2(\eta)\,(N-1)$ equals the average number of clumps of exceedances above η, where a clump of exceedances is defined as a maximum number of consecutive exceedances above η.

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A reinterpretation of this is that $\hat{\varepsilon}_2(\eta)\,(N-1)$ equals the average number of clumps of exceedances above η, where a clump of exceedances is defined as a maximum number of consecutive exceedances above η.

In general, $\hat{\varepsilon}_k(\eta)\,(N-1)$ then equals the average number of clumps of exceedances above η separated by at least $k-1$ non-exceedances.

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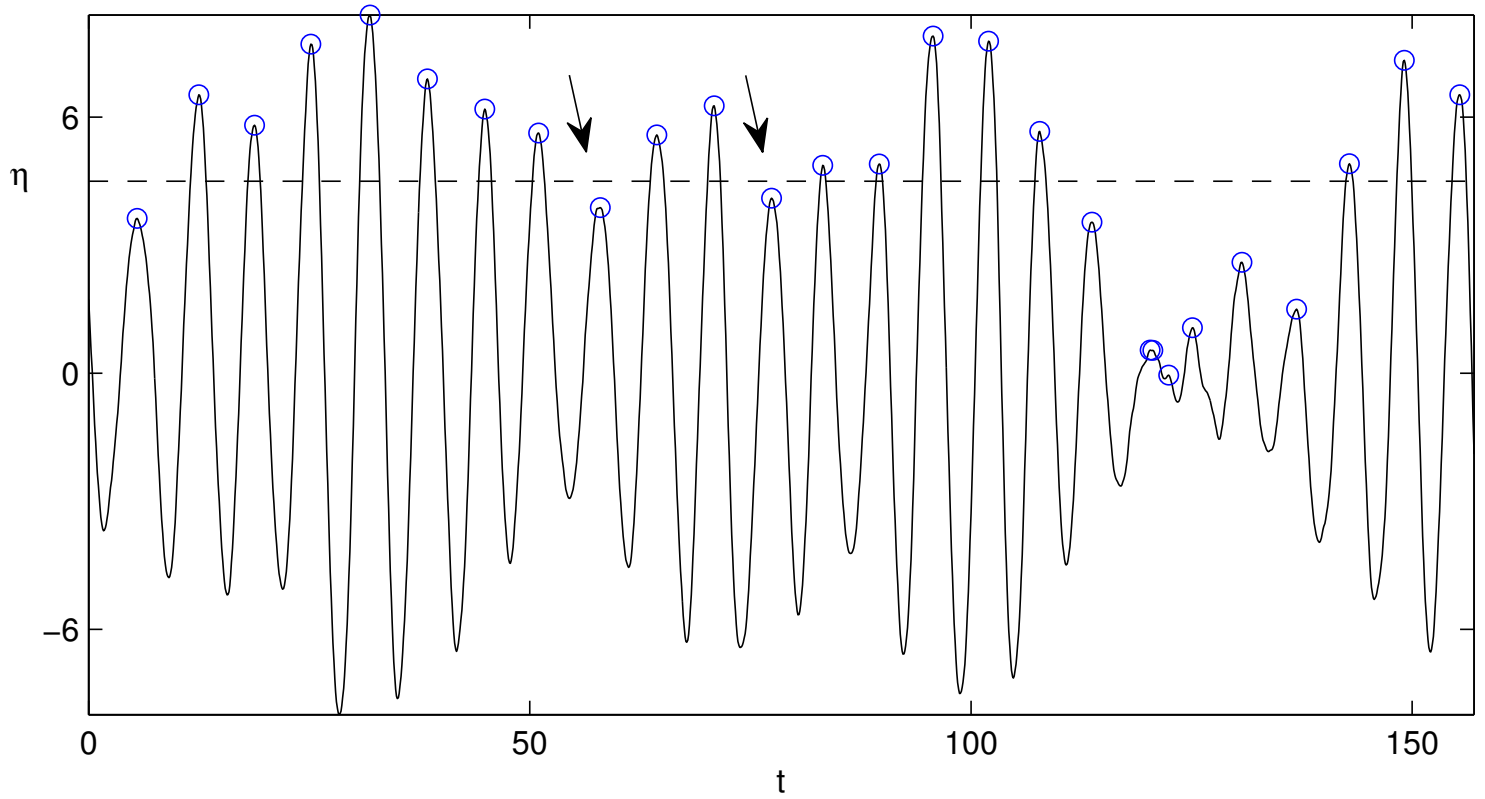
Empirical Estimation of ACER



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Parametric form of the ACER

The relevant asymptotic extreme value distribution is assumed to be of Gumbel type. Using the asymptotic form as a guide, it is assumed that

$$\tilde{\varepsilon}_k(\eta) \approx q_k(\eta)\ \exp\{-a_k(\eta - b_k)^{c_k}\}\,,\ \ \eta \geq \eta_1\,,$$

where the function $q_k(\eta)$ is slowly varying compared with the exponential function $\exp\{-a_k(\eta - b_k)^{c_k}\}$ and a_k, b_k, and c_k are suitable constants.

Parametric form of the ACER

The relevant asymptotic extreme value distribution is assumed to be of Gumbel type. Using the asymptotic form as a guide, it is assumed that

$$\tilde{\varepsilon}_k(\eta) \approx q_k(\eta)\ \exp\{-a_k(\eta - b_k)^{c_k}\}\,,\ \ \eta \geq \eta_1\,,$$

where the function $q_k(\eta)$ is slowly varying compared with the exponential function $\exp\{-a_k(\eta - b_k)^{c_k}\}$ and a_k, b_k, and c_k are suitable constants.

Note that the values $c_k = q_k = 1$ correspond to the asymptotic case.

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Optimized Fit of Parameters

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The choice of parameters a, b, c, q can be made by optimizing the fit on the log level.

We use Levenberg-Marquardt least squares optimization. The mean square error function to be minimized is written as

$$F(q,a,b,c) = \sum_{j=1}^{N} w_j \left(\log \hat{\varepsilon}_k(\eta_j) - \log q + a(\eta_j - b)^c \right)^2 ,$$

where $w_j = \left(\log CI^+(\eta_j) - \log CI^-(\eta_j) \right)^{-2}$, and
$CI^{\pm}(\eta) = \hat{\varepsilon}_k(\eta) \pm 1.96\, \hat{s}_k(\eta)/\sqrt{R}$ (95% conf. int.)

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Optimized Fit of Parameters

Let $y_j = \log \hat{\varepsilon}_k(\eta_j)$, $x_j = x_j(b, c) = (\eta_j - b)^c$. Then

$$F(q, a, b, c) = \sum_{j=1}^{N} w_j \left(y_j - \log q + a x_j \right)^2 ,$$

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$$F(q,a,b,c) = \sum_{j=1}^{N} w_j \left(y_j - \log q + a x_j\right)^2 ,$$

Hence, for fixed b and c, we have a weighted linear regression problem with solutions:

$$a^*(b,c) = -\frac{\sum_{j=1}^{N} w_j (x_j - \overline{x})(y_j - \overline{y})}{\sum_{j=1}^{N} w_j (x_j - \overline{x})^2} ,$$

and

$$\log q^*(b,c) = \overline{y} + a^*(b,c)\overline{x} ,$$

where $\overline{x} = \sum_{j=1}^{N} w_j x_j / \sum_{j=1}^{N} w_j$, with a similar definition of $\overline{y}$.

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Optimized Fit of Parameters

We may now use the Levenberg-Marquardt method on the function $\tilde{F}(b,c) = F(q^*(b,c), a^*(b,c), b, c)$ to find the optimal values b^* and c^*, and then use the expressions for $a^*(b,c)$ and $\log q^*(b,c)$ to calculate the corresponding global optimal values a^* and q^*.

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For estimation of a confidence interval for the predicted extreme value provided by the optimal curve, the empirical confidence band is reanchored to the optimal curve. The optimally fitted curves to the boundaries of the reanchored confidence band will determine an optimized confidence interval on the predicted extreme value.

Nonstationary Time Series

To deal with nonstationary time series, it is recognized that $\mathrm{E}[B_{kj}(\eta)] \to 1$ when $\eta \to \infty$.

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Hence,

$$P(\eta) \approx \exp\Big(-\sum_{j=k}^{N} \alpha_{kj}(\eta)\Big) = \exp\Big(-\sum_{j=k}^{N} \frac{\mathrm{E}[A_{kj}(\eta)]}{\mathrm{E}[B_{kj}(\eta)]}\Big)$$

$$\underset{\eta\to\infty}{\simeq} \exp\Big(-\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)]\Big) = \exp\Big(-(N+k-1)\tilde{\varepsilon}_k(\eta)\Big)$$

where the modified ACER function $\tilde{\varepsilon}_k(\eta)$ is given as

$$\tilde{\varepsilon}_k(\eta) = \frac{\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)]}{N-k+1}.$$

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Nonstationary Time Series

Assume that the time series can be segmented into K blocks such that $\mathrm{E}[A_{kj}(\eta)]$ remains approximately constant within each block.

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Assume that $\sum_{j \in C_i} \mathrm{E}[A_{kj}(\eta)] \approx \sum_{j \in C_i} a_{kj}(\eta)$ for a sufficient range of η-values, where C_i denotes the set of indices for block no. i, $i = 1, \ldots, K$,

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Then $\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)] \approx \sum_{j=k}^{N} a_{kj}(\eta)$.

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Then $\sum_{j=k}^{N} \mathrm{E}[A_{kj}(\eta)] \approx \sum_{j=k}^{N} a_{kj}(\eta)$.

Thus, seasonal effects are automatically accounted for by the modified ACER method.

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Long Term Extremes

If the whole time series over a long term scenario is available, we have already shown that the long term statistics using ACER functions may be estimated directly from the time series.

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However, in many cases it would be more practical to analyze each short term condition separately and combine the obtained ACER functions after that.

Long Term Extremes

If the whole time series over a long term scenario is available, we have already shown that the long term statistics using ACER functions may be estimated directly from the time series.

However, in many cases it would be more practical to analyze each short term condition separately and combine the obtained ACER functions after that.

This would, e.g. be the typical approach in a simulation based long term statistical analysis where the short term response time series would be simulated and the resulting time series subjected to an ACER analysis.

Long Term Extremes

A long term formulation is obtained by considering that there are m, say, short term conditions. Assume that the number of data in condition no.j is N_j, $j = 1, \ldots, m$, and $N = \sum_{j=1}^{m} N_j$. Then we may write,

$$\hat{\varepsilon}_k(\eta) = \sum_{j=1}^{m} \hat{\varepsilon}_k^{(j)}(\eta) \; \frac{N_j - k + 1}{N - k + 1} \, ,$$

where the ACER function $\hat{\varepsilon}_k^{(j)}(\eta)$ is estimated for condition no. j.

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So, we obtain the long term extreme value distribution as,

$$P(\eta) \approx \exp\big(- (N - k + 1)\hat{\varepsilon}_k(\eta)\big) \, .$$

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Scatter Diagram North Sea, 1973 – 2001

h_s (m)	t_p(s)																		
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**> 20**
0.5	18	15	123	113	110	390	260	91	38	42	32	3	19	13	9	1	3	2	7
1.0	16	49	675	433	589	1442	1802	959	273	344	125	33	64	29	13	1	7	1	6
1.5	5	32	417	893	1107	1486	2757	1786	636	731	299	121	92	43	18	10	5	2	13
2.0	1	0	102	741	1290	1496	2575	1968	780	868	492	200	116	51	31	8	4	4	8
2.5	0	0	9	256	969	1303	2045	1892	803	941	484	181	157	58	23	19	5	1	8
3.0	0	0	1	45	438	1029	1702	1898	705	957	560	218	196	92	40	11	4	2	5
3.5	0	0	1	4	124	650	1169	1701	647	865	456	237	162	100	36	12	6	1	5
4.0	0	0	2	0	33	270	780	1369	573	868	427	193	157	91	51	13	3	0	1
4.5	0	0	0	0	3	90	459	1017	466	761	380	127	137	86	31	23	6	5	0
5.0	0	0	0	0	0	15	228	647	408	737	354	119	96	50	32	18	2	4	1
5.5	0	0	0	0	0	2	68	337	363	580	283	94	92	31	24	10	6	2	0
6.0	0	0	0	0	0	1	20	166	221	418	307	63	76	24	13	9	4	0	0
6.5	0	0	0	0	0	0	5	50	140	260	257	59	49	20	12	4	2	2	2
7.0	0	0	0	0	0	0	0	23	90	180	193	41	53	20	5	3	3	0	0
7.5	0	0	0	0	0	0	0	6	25	93	121	45	46	17	5	5	0	1	0
8.0	0	0	0	0	0	0	0	3	14	50	84	26	47	11	6	0	1	0	0
8.5	0	0	0	0	0	0	0	0	7	25	45	23	25	20	8	0	0	0	0
9.0	0	0	0	0	0	0	0	1	2	12	30	22	20	19	0	0	0	0	0
9.5	0	0	0	0	0	0	0	0	1	2	20	21	14	7	1	1	0	1	0
10.0	0	0	0	0	0	0	0	0	0	2	5	4	21	6	2	0	0	0	0
10.5	0	0	0	0	0	0	0	0	0	3	4	8	9	12	2	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0	0	2	0	4	3	1	0	1	0	0
11.5	0	0	0	0	0	0	0	0	0	0	2	1	2	3	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	0	0	0	0
12.5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

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Long Term Extremes

With reference to a scatter diagram, an alternative equivalent formulation is obtained. Assume that the number of sea states in condition (i, j) is N_{ij}, $i = 1, \ldots, m$ and $j = 1, \ldots, n$, and $N = \sum_{i=1}^{m} \sum_{j=1}^{n} N_{ij}$. Then,

$$\hat{\varepsilon}_k(\eta) = \sum_{i=1}^{m} \sum_{j=1}^{n} \hat{\varepsilon}_k^{(ij)}(\eta) \ \frac{N_{ij}}{N} \,,$$

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So, again we obtain the long term extreme value distribution as,

$$P(\eta) \approx \exp\big(-(N - k + 1)\hat{\varepsilon}_k(\eta)\big) \, .$$

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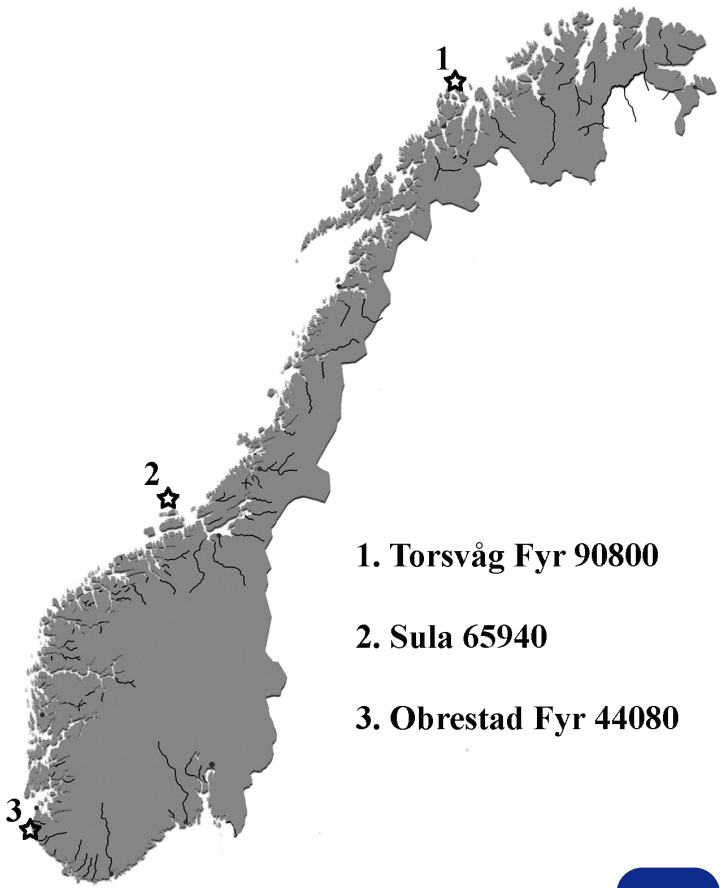
Wind speed statistics

Locations of wind speed measurements



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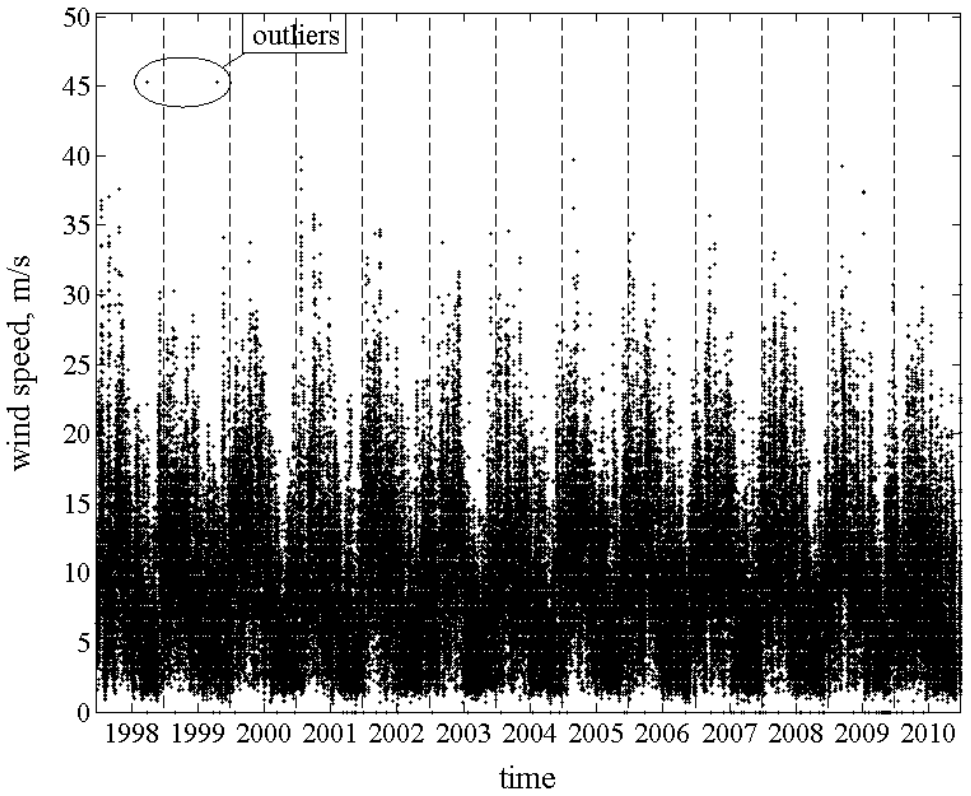
Wind speeds at Torsvåg Lighthouse

Time series over 13 years of hourly maxima of gust wind.



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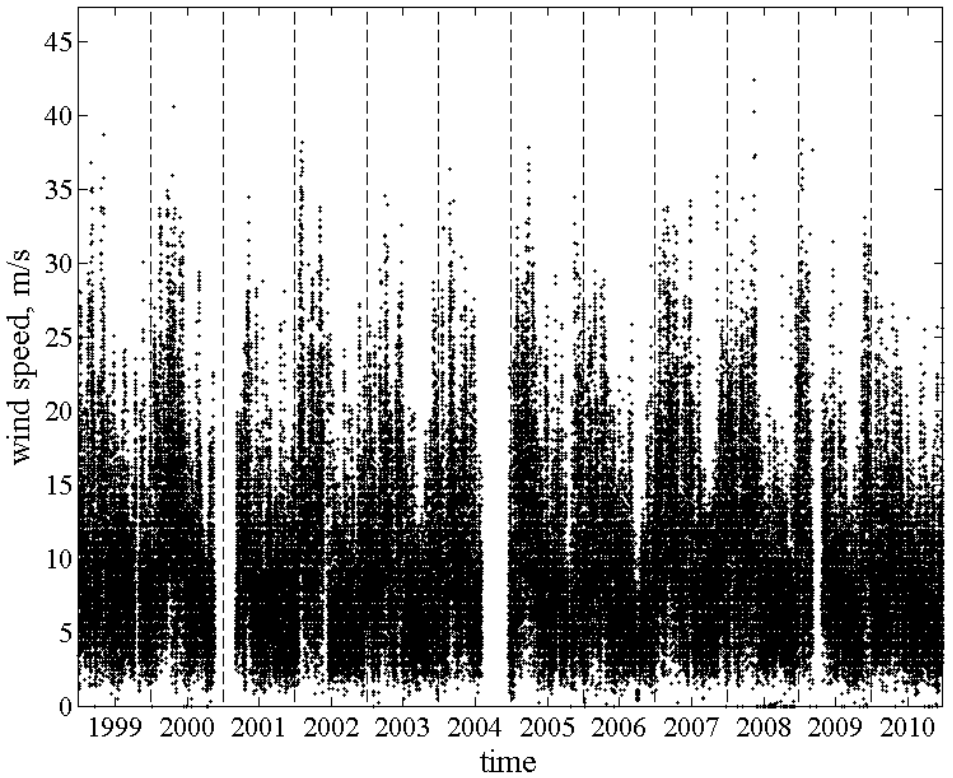
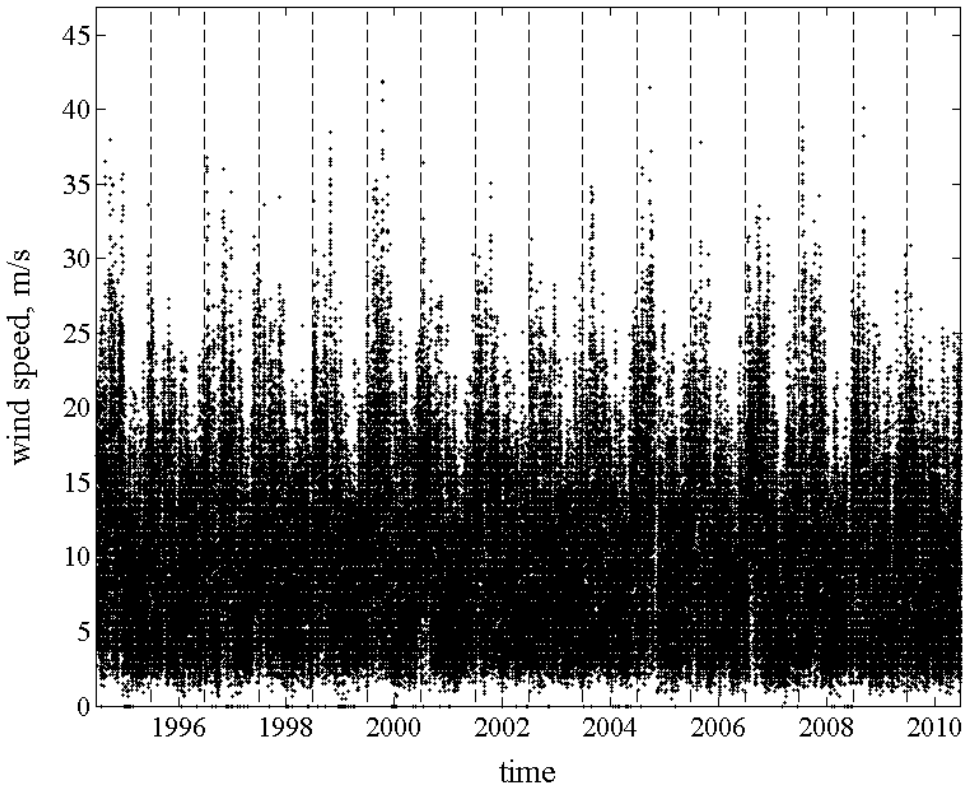
Wind speeds at Sula Lighthouse

Time series over 12 years of hourly maxima of gust wind.



Wind speeds at Obrestad Lighthouse

Time series over 16 years of hourly maxima of gust wind.



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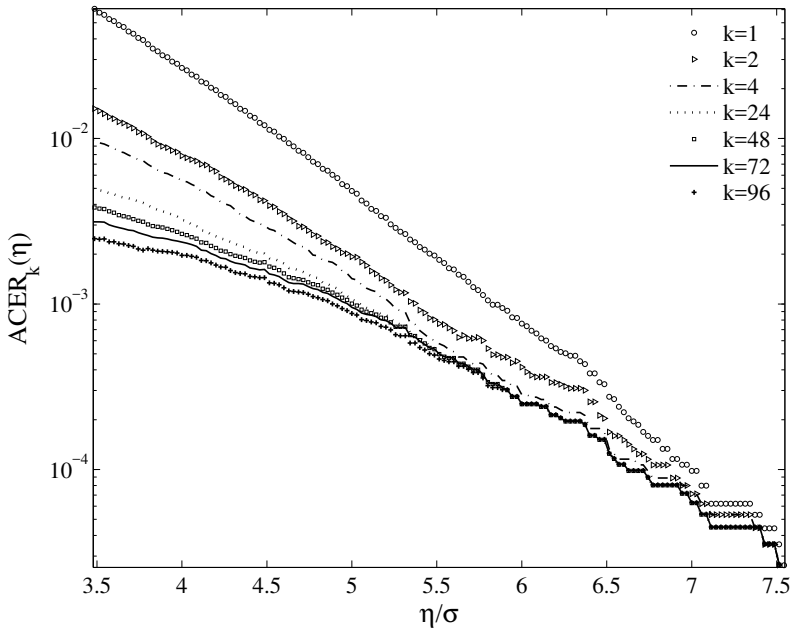
Torsvåg Lighthouse wind speed statistics

The ACER estimates for different degrees of conditioning.



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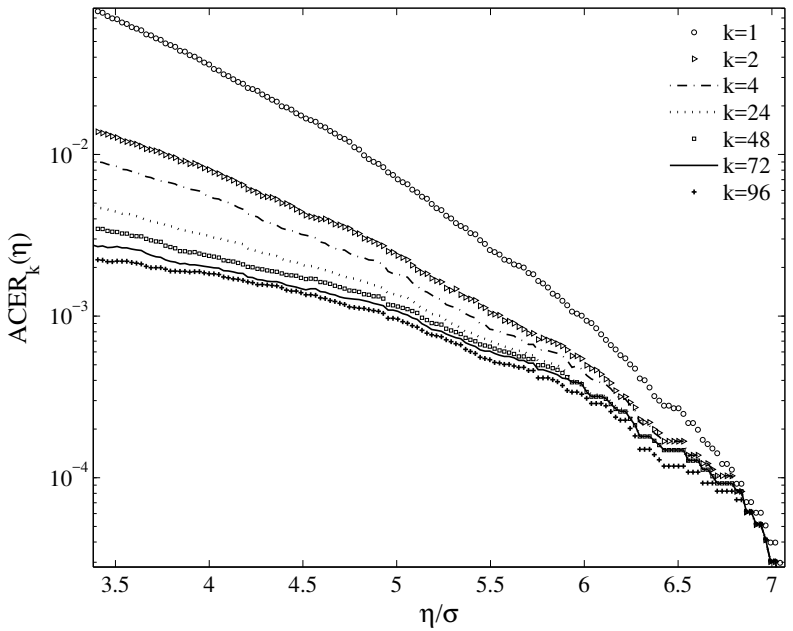
Sula wind speed statistics

The ACER estimates for different degrees of conditioning.



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Obrestad Lighthouse wind speed statistics

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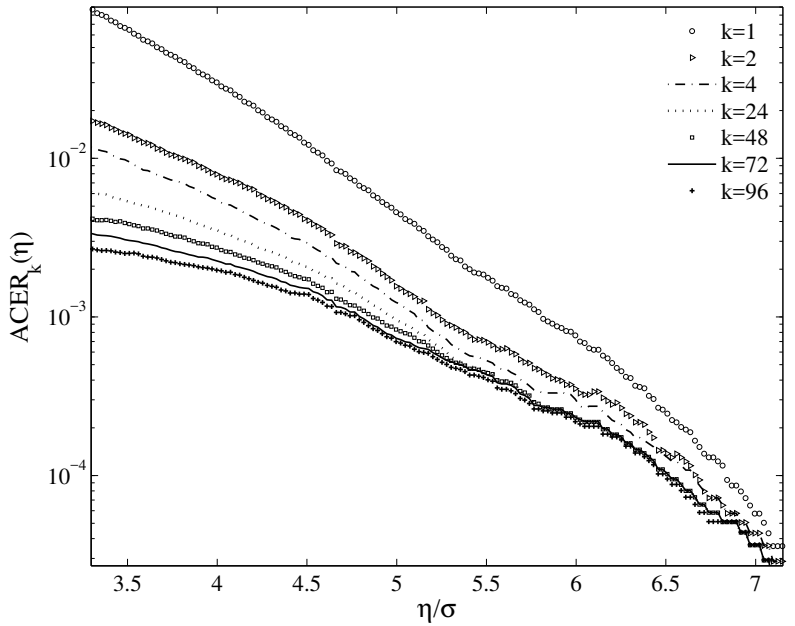

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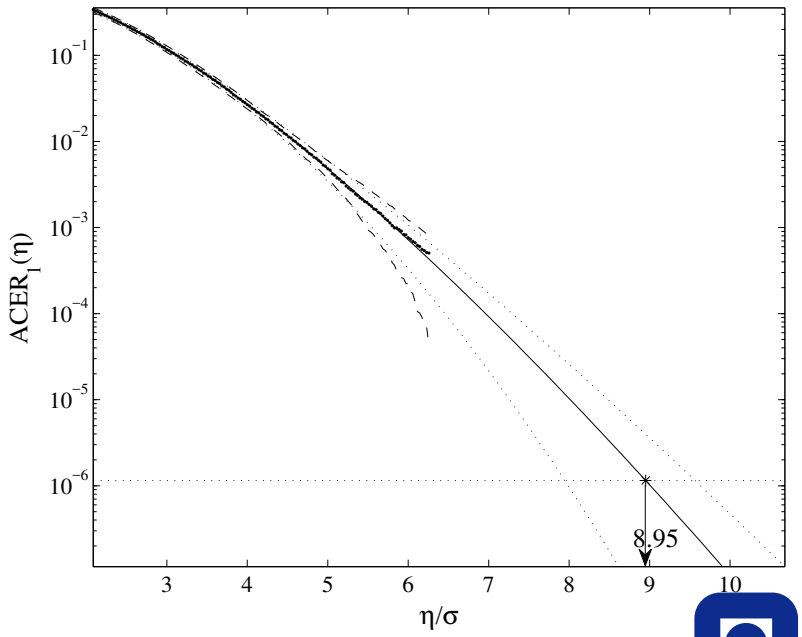
Torsvåg Lighthouse wind speed statistics

13 years hourly maximum data. $\hat{\varepsilon}_1(\eta)$ (*); Optimized curve fit (—); Empirical 95% confidence band (- -); Optimized confidence band ($\cdot\ \cdot\ \cdot$); Predicted 100 year return level estimate = 47.46 m/s and 95% CI = (42.11, 50.71) m/s.



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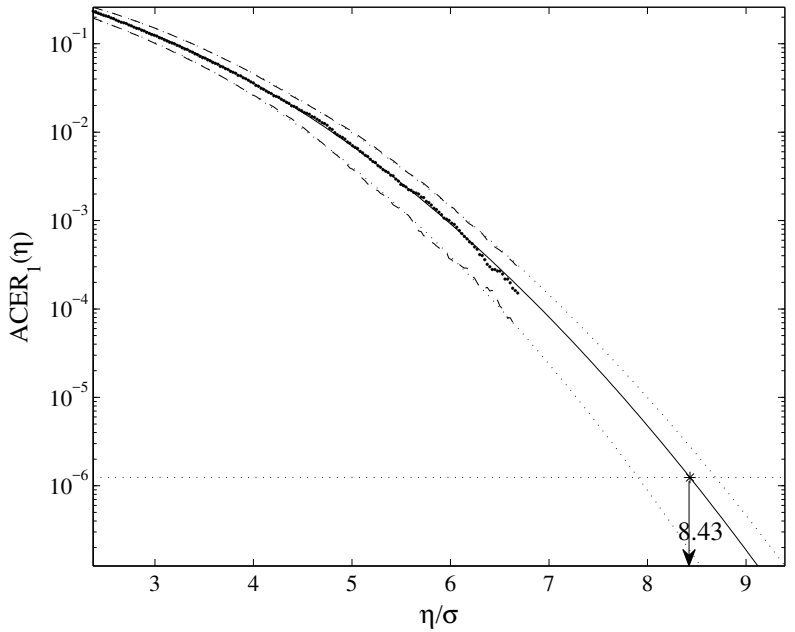
Sula wind speed statistics

12 years hourly maximum data. $\hat{\varepsilon}_1(\eta)$ (*); Optimized curve fit (—); Empirical 95% confidence band (- -); Optimized confidence band ($\cdot\ \cdot\ \cdot$); Predicted 100 year return level estimate = 46.33 m/s and 95% CI = (43.41, 47.77) m/s.



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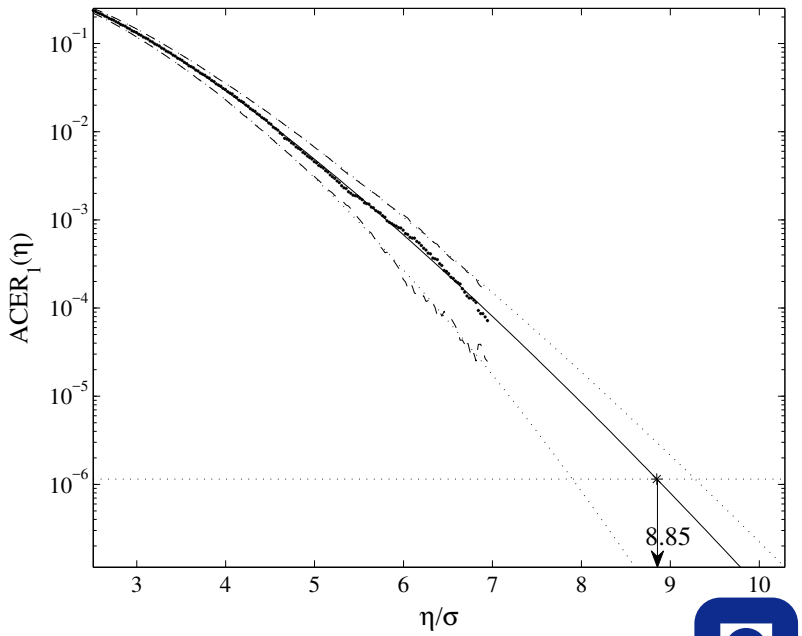
Obrestad Lighthouse wind speed statistics

16 years hourly maximum data. $\hat{\varepsilon}_1(\eta)$ (*); Optimized curve fit (—); Empirical 95% confidence band (- -); Optimized confidence band ($\cdot\ \cdot\ \cdot$); Predicted 100 year return level estimate = 48.38 m/s and 95% CI = (43.18, 50.74) m/s.



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Narrow Band Process

In engineering mechanics, a classical extreme response prediction problem is the case of a lightly damped mechanical oscillator subjected to random forces.

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$$\ddot{X}(t) + 2\zeta\omega_e\dot{X}(t) + \omega_e^2 X(t) = W(t)\,,$$

where ζ = relative damping, ω_e = undamped eigenfrequency, and $W(t)$ = a stationary Gaussian white noise (of suitable intensity).

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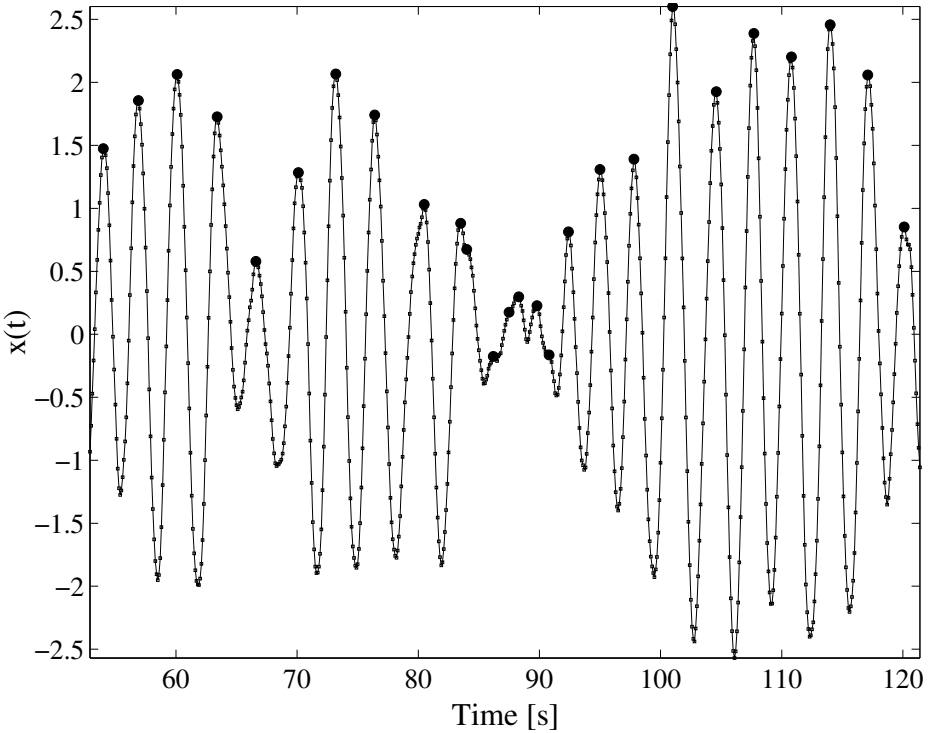
where ζ = relative damping, ω_e = undamped eigenfrequency, and $W(t)$ = a stationary Gaussian white noise (of suitable intensity). For small values of ζ, the response time series will exhibit narrow band characteristics. This manifests itself by producing a strong beating of the response time series, which means that the size of the response peaks will change slowly in time,

Narrow Band Process



Part of the narrow-band response time series of the linear oscillator with fully sampled and peak values indicated.

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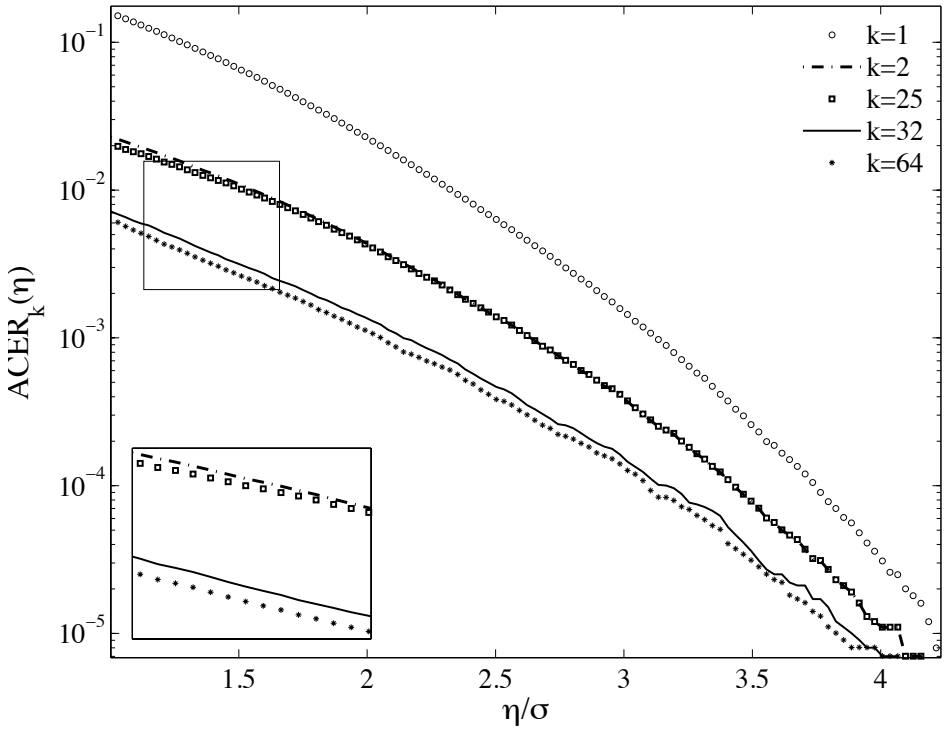
Narrow Band Process



Comparison between ACER estimates for different degrees of conditioning for the narrow-band time series.

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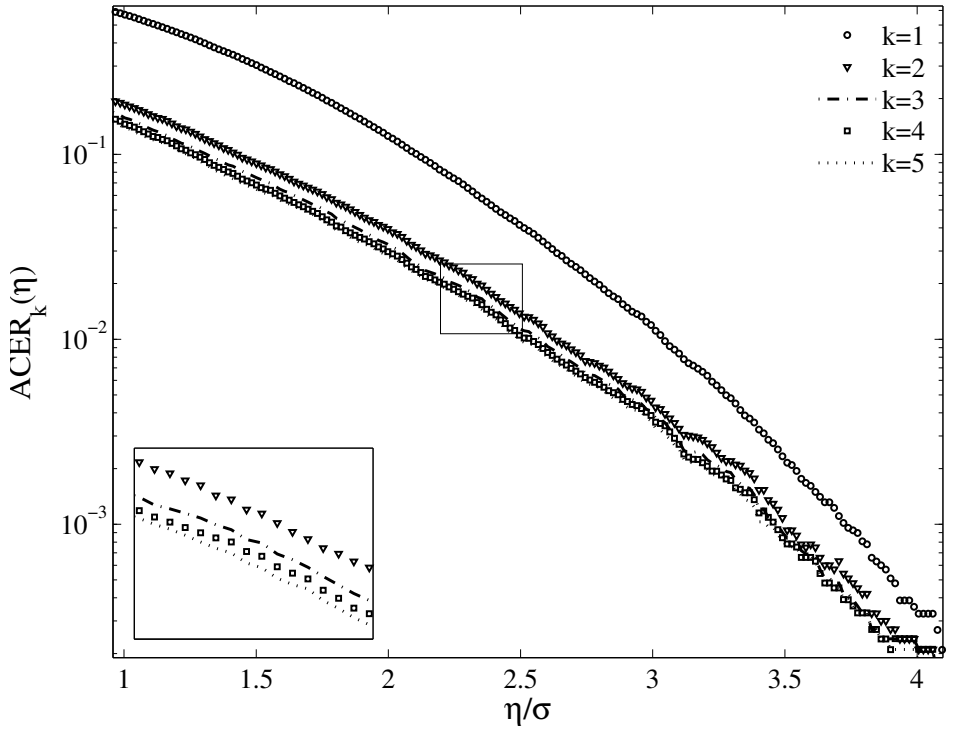
Narrow Band Process



Comparison between ACER estimates for different degrees of conditioning based on the time series of the peak values.

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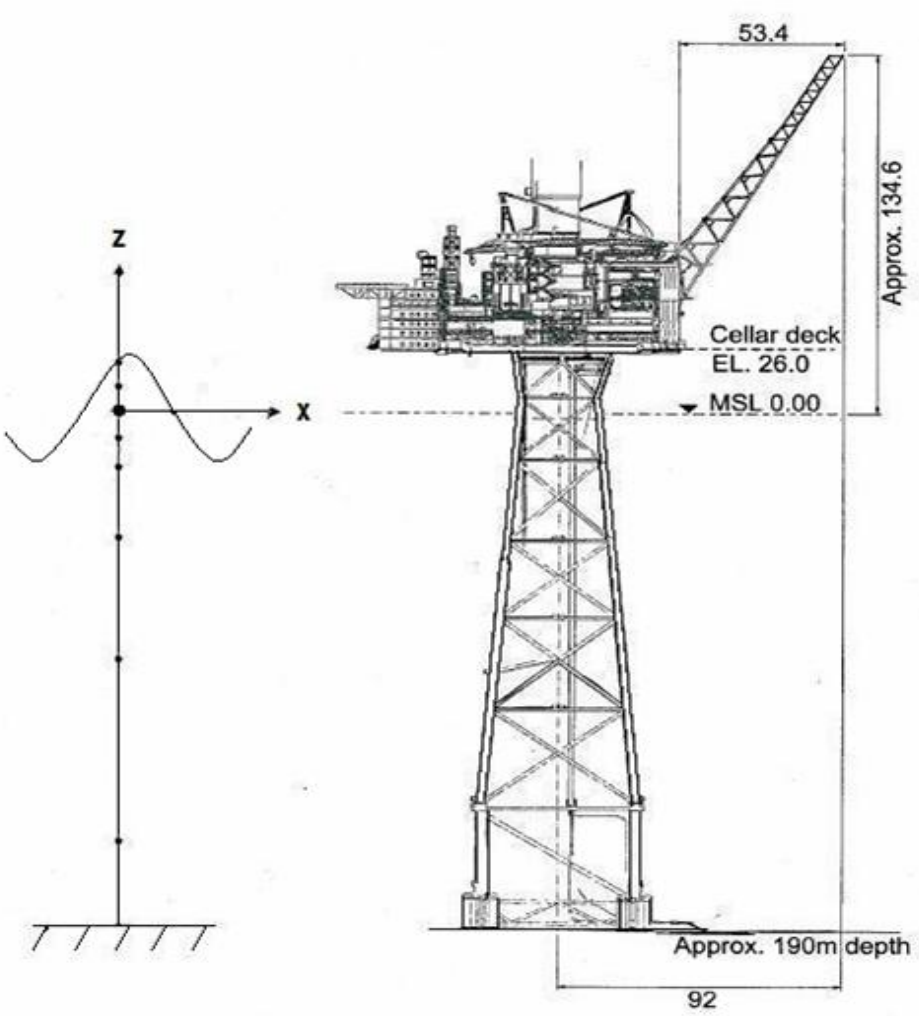
The Kvitebjorn Jacket Platform

The Kvitebjørn jacket platform



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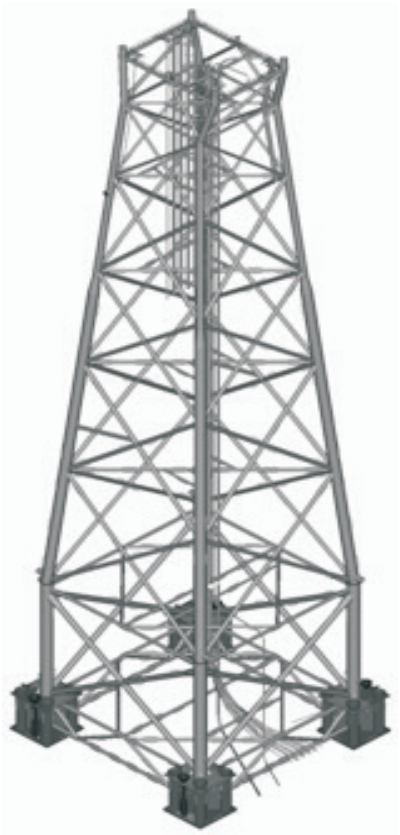
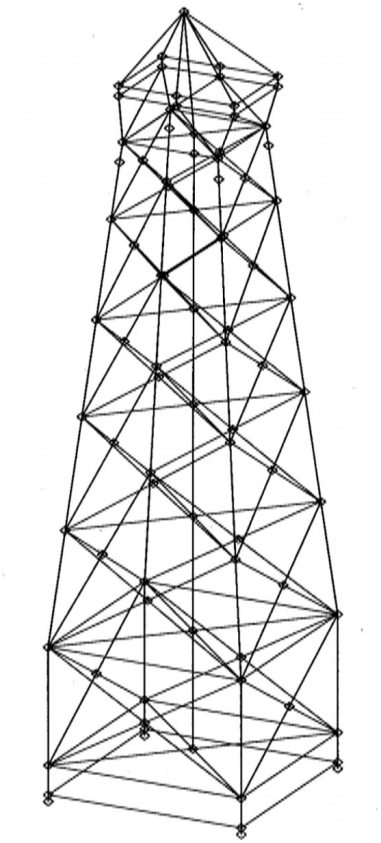
The Kvitebjorn Jacket Platform

The Kvitebjørn jacket platform with the superstructure removed.





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The Kvitebjorn Jacket Platform

— The equation of motion for the horizontal excursions of the jacket at main deck level is

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{Q}.$$

The Kvitebjorn Jacket Platform

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— $\mathbf{X} = (X_1, \ldots, X_N)^T$ where $X_k = X_k(t)$, $k = 1, \ldots, N$, denote displacement of the k-th node $\mathbf{x}_k = (x_k, y_k, z_k)$ in the wave direction, which is the positive x-direction.

The Kvitebjorn Jacket Platform

— The equation of motion for the horizontal excursions of the jacket at main deck level is

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{Q}.$$

— $\mathbf{X} = (X_1, \ldots, X_N)^T$ where $X_k = X_k(t)$, $k = 1, \ldots, N$, denote displacement of the k-th node $\mathbf{x}_k = (x_k, y_k, z_k)$ in the wave direction, which is the positive x-direction.

— $\mathbf{Q} = (Q(t, \mathbf{x}_1), \ldots, Q(t, \mathbf{x}_N))^T$, where $Q(t, \mathbf{x}_k) = F_{in}(t, \mathbf{x}_k) + F_d(t, \mathbf{x}_k)$, $k = 1, \ldots, N$ and $-d = z_1 \leq z_k \leq z_N = L - d$, where $d = 190$ m is the water depth and $L = 216$ m is the jacket support height.

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The Kvitebjorn Jacket Platform

— The inertia force components are given as

$$F_{in}(t, \mathbf{x}_k) = k_m \, \dot{U}(t, \mathbf{x}_k)$$

The Kvitebjorn Jacket Platform

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$$F_{in}(t, \mathbf{x}_k) = k_m \dot{U}(t, \mathbf{x}_k)$$

— The drag force components

$$F_d(t, \mathbf{x}_k) = k_d \left(U(t, \mathbf{x}_k) + U_c\right) |U(t, \mathbf{x}_k) + U_c|$$

The Kvitebjorn Jacket Platform

— The inertia force components are given as

$$F_{in}(t, \mathbf{x}_k) = k_m \dot{U}(t, \mathbf{x}_k)$$

— The drag force components

$$F_d(t, \mathbf{x}_k) = k_d \left(U(t, \mathbf{x}_k) + U_c\right)|U(t, \mathbf{x}_k) + U_c|$$

—

$$k_m = C_m \rho \pi D^2/4, \; k_d = C_d \rho D/2$$

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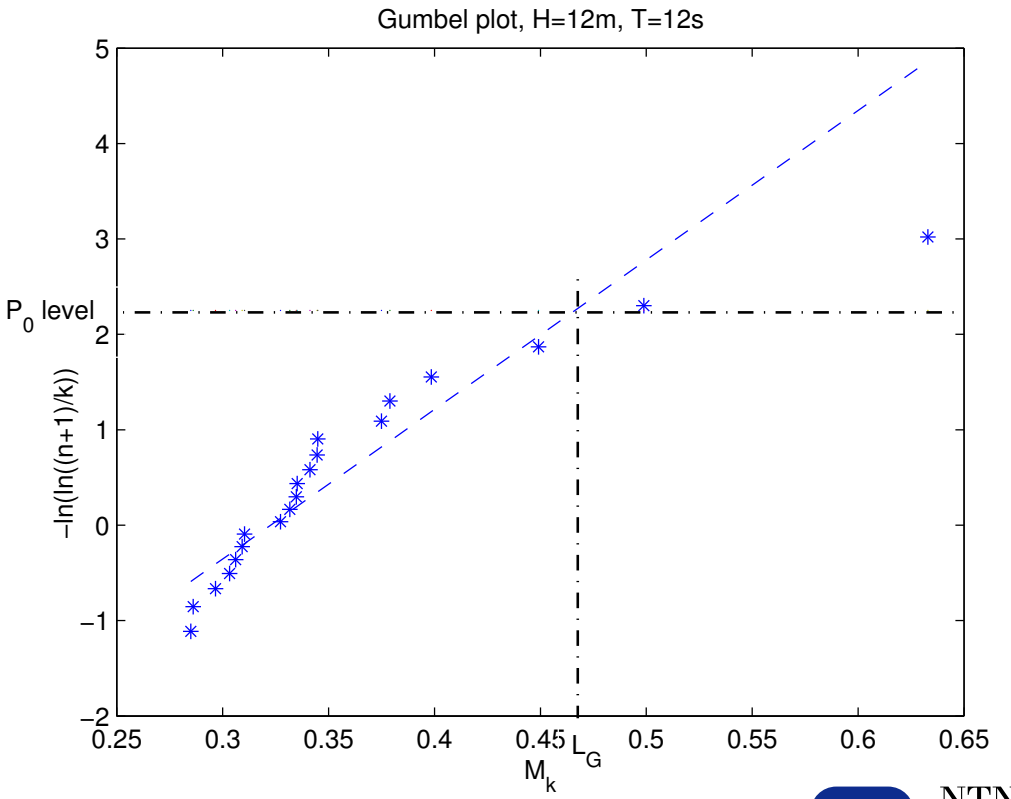
Gumbel plot of 20 simulated 3 hour extremes with fitted Gumbel distribution. Sea state with $H_s = 12$ m, $T_p = 12$ s.



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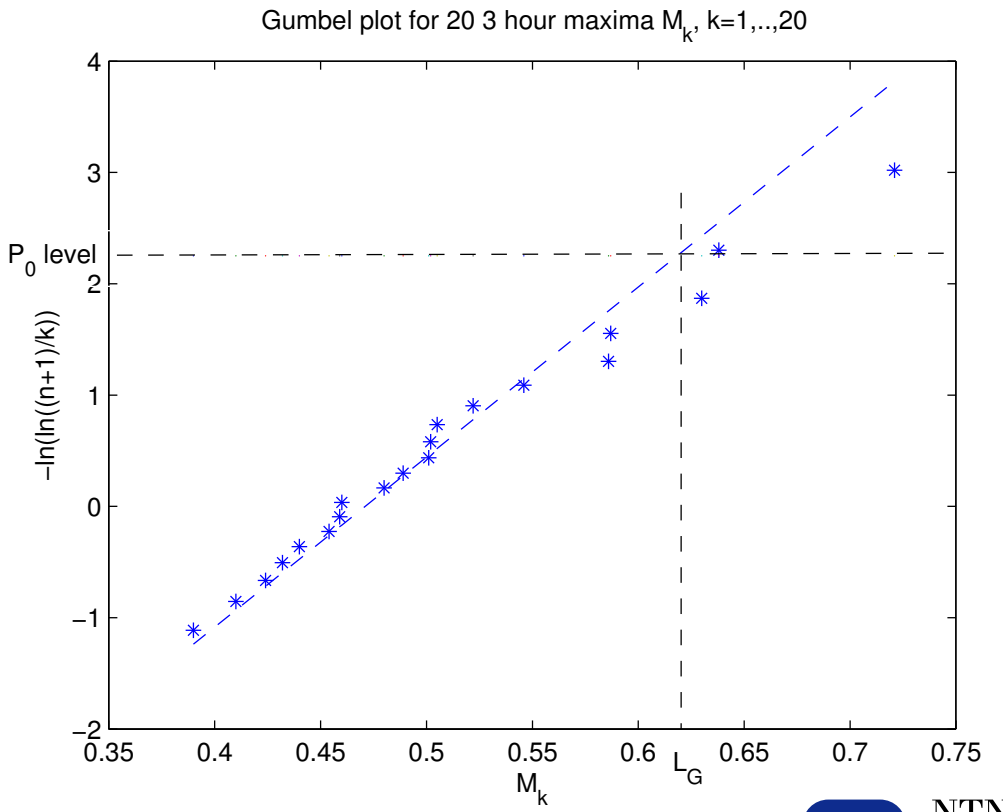
Gumbel plot of 20 simulated 3 hour extremes with fitted Gumbel distribution. Sea state with $H_s = 14.7$ m, $T_p = 15$ s.



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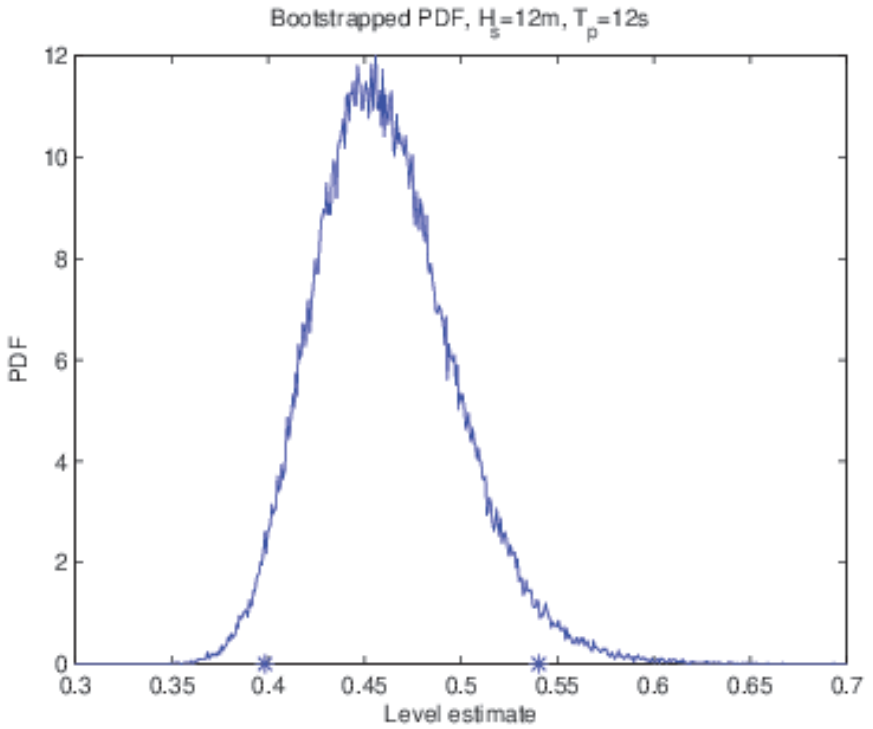
Empirical density of the predicted 90% fractile value based on sample of size 20 for the sea state with $H_s = 12$ m, $T_p = 12$ s. The $*$ indicates the limits of $\text{CI}_{0.95}$.



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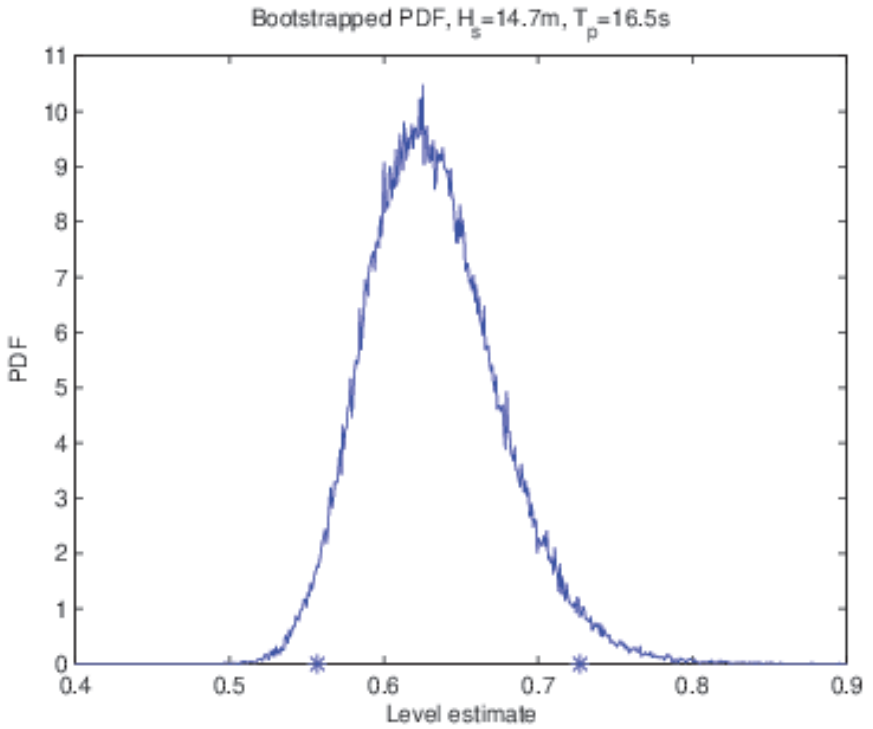
Empirical density of the predicted 90% fractile value based on sample of size 20 for the sea state with $H_s = 14.7$ m, $T_p = 16.5$ s. The $*$ indicates the limits of $\text{CI}_{0.95}$.



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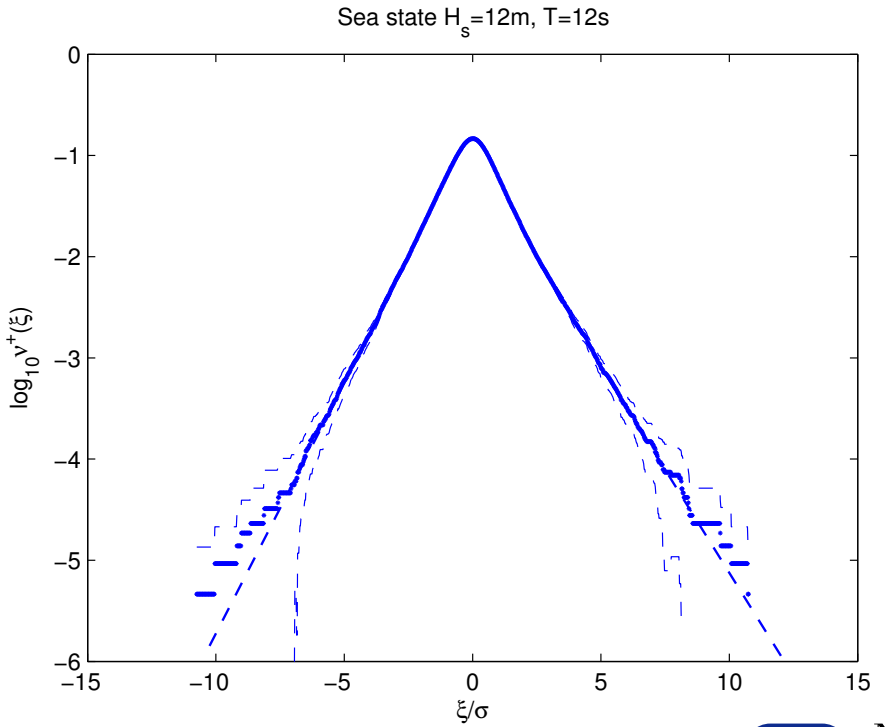
The ACER function ε_2 (mean upcrossing rate) along with 95% confidence bands $(--)$ for the sea state with $H_s = 12$ m, $T_p = 12$ s, $\sigma = 0.047$ m. $*$: Monte Carlo; $- - -$: linear fit.



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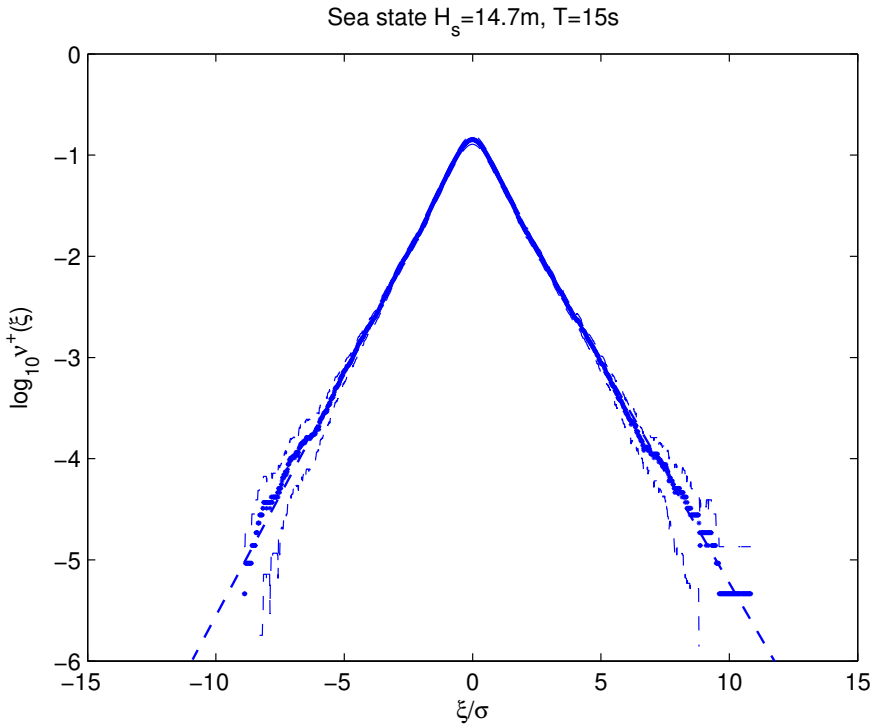
The ACER function ε_2 (mean upcrossing rate) with 95% confidence bands $(--)$ for the sea state with $H_s = 14.7$ m, $T_p = 15$ s, $\sigma = 0.066$ m. $*$: Monte Carlo; $- - -$: linear fit.



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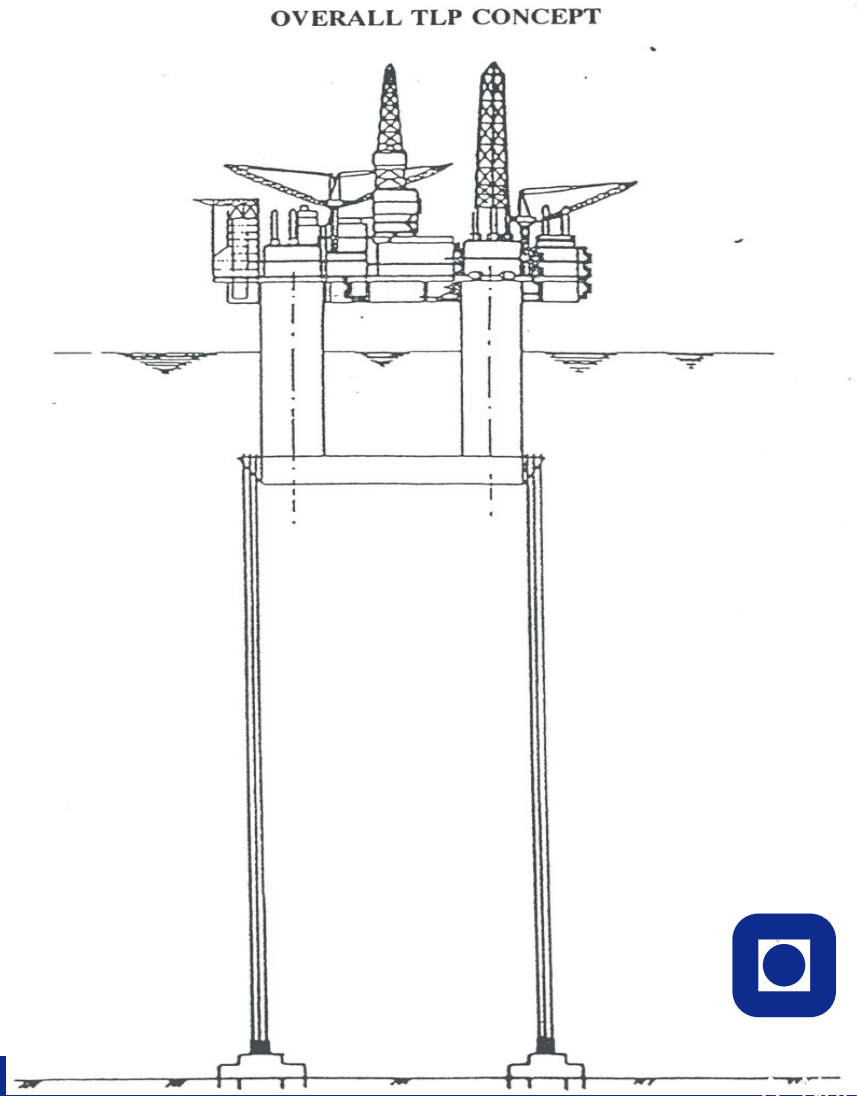
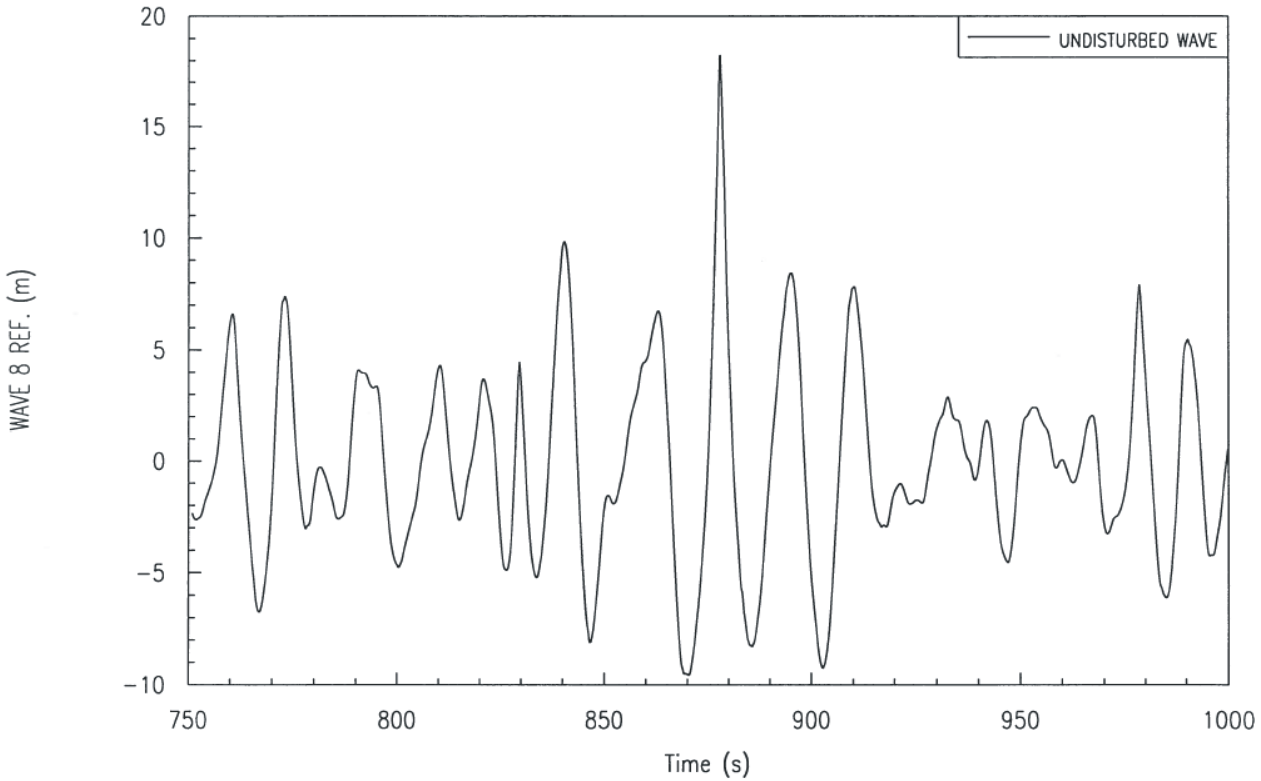
The Heidrun TLP

Heidrun TLP as seen from the side.



Waves

Time series of wave elevation.



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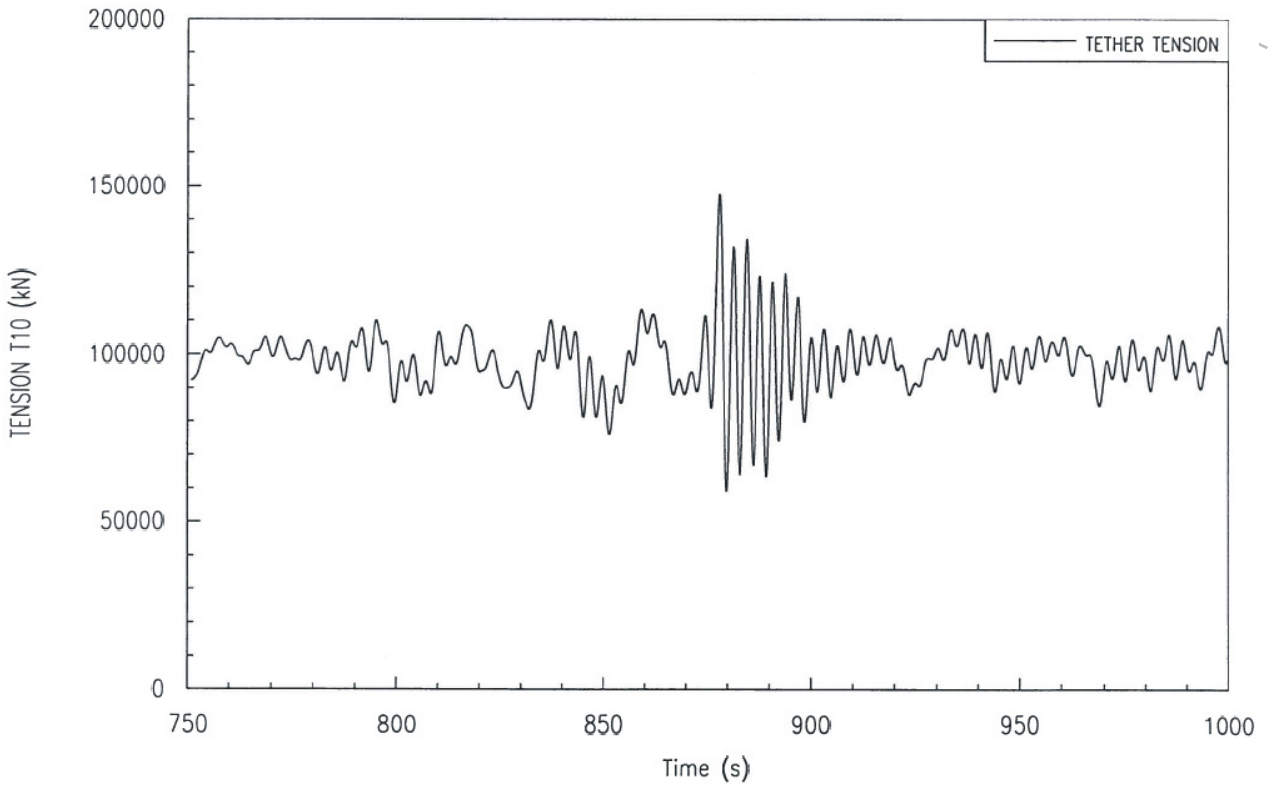
Tether tension

Time series of tether tension T10, with a ringing event caused by a steep wave.



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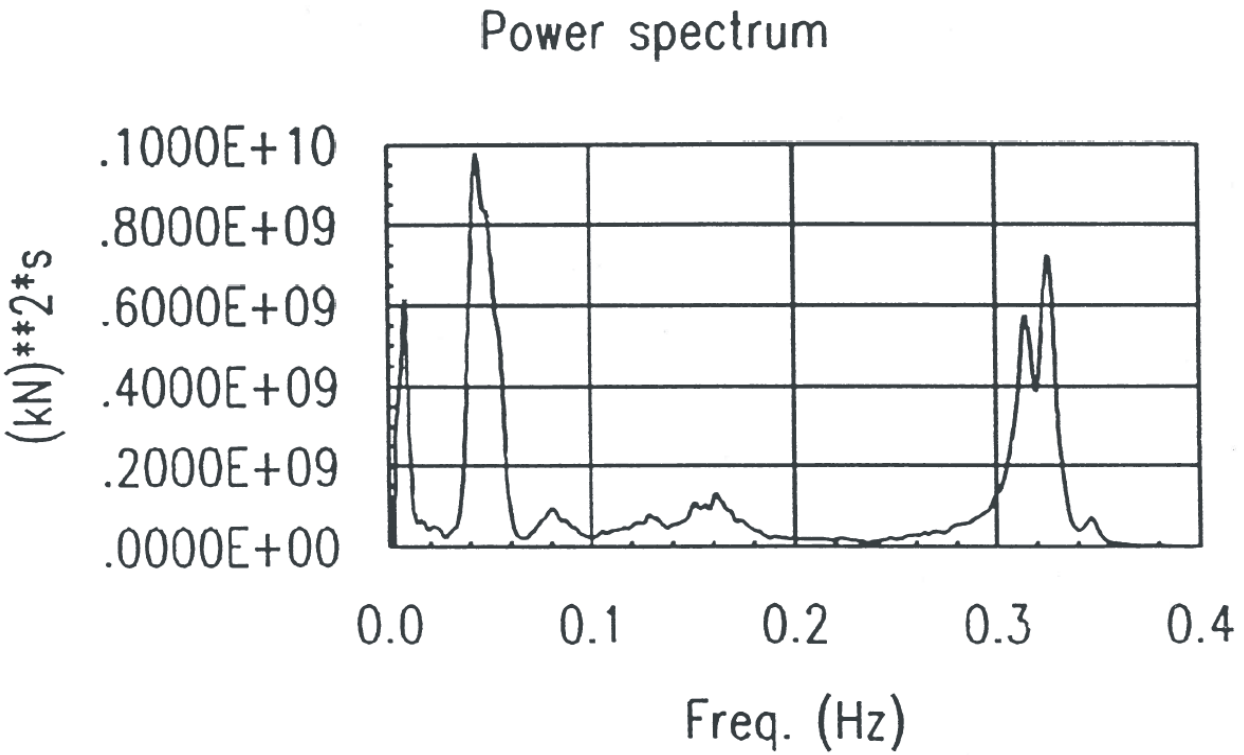
Power spectrum

Power spectrum of tension in tether T10.



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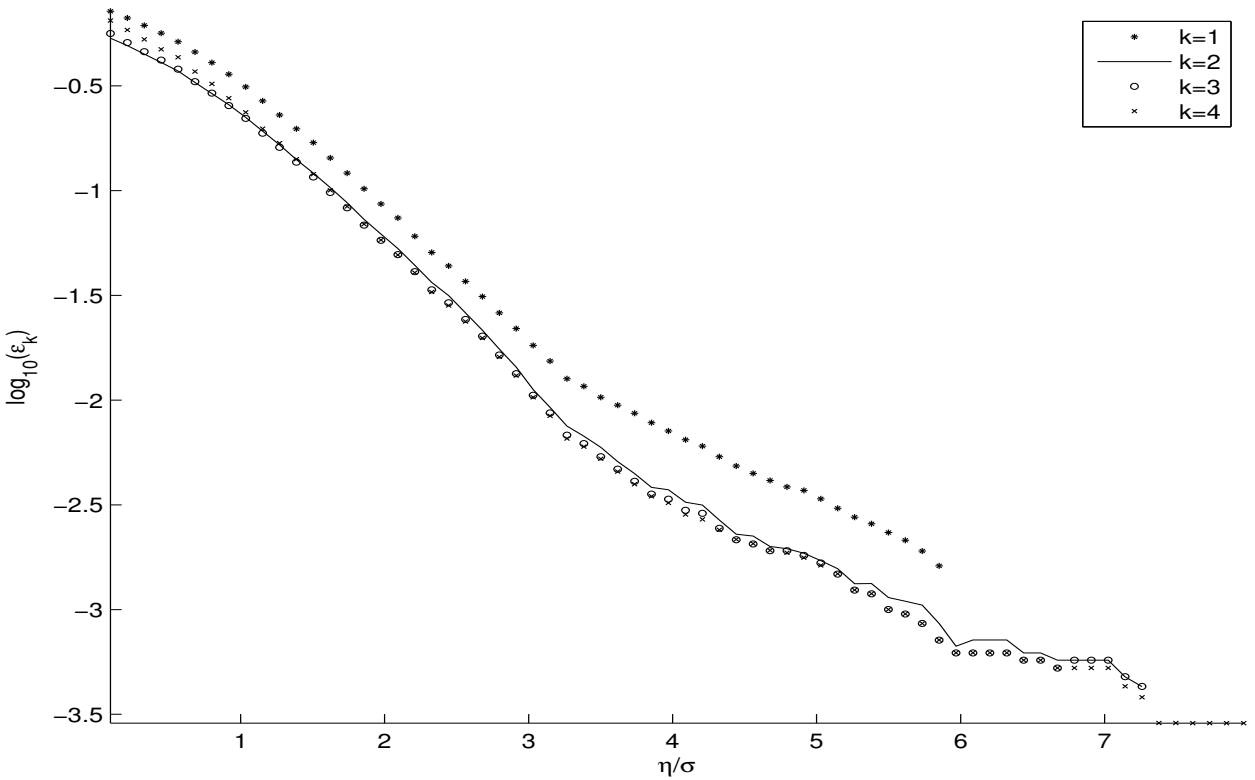
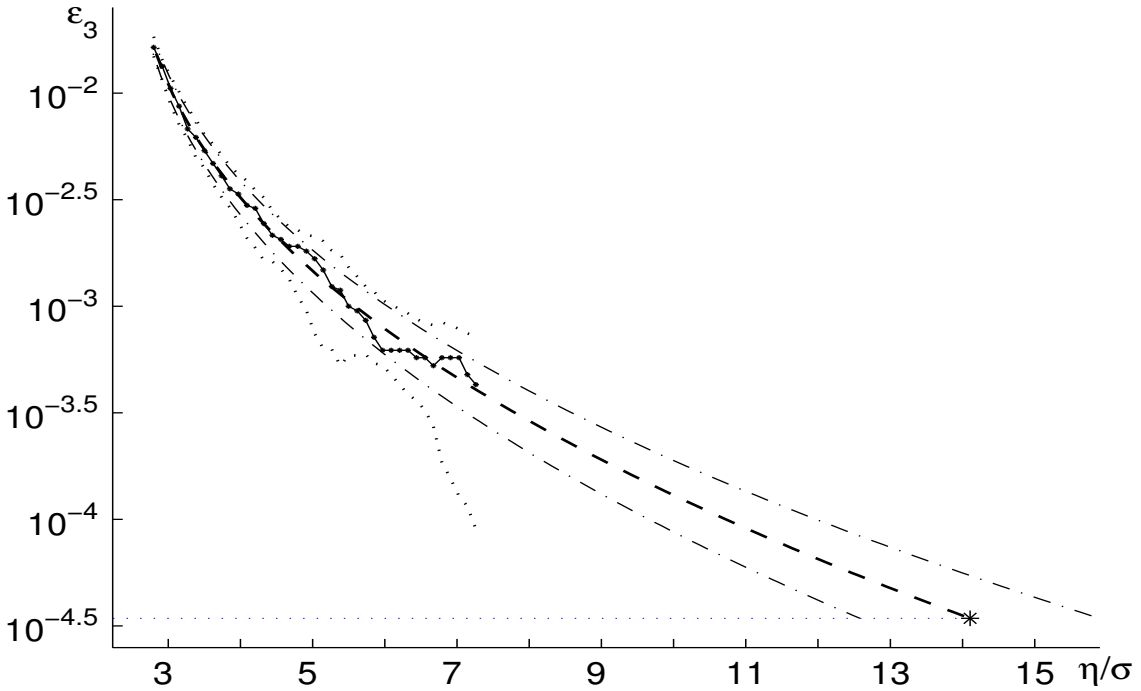
ACER

Log plot of ACER $\varepsilon_k(\eta)$, sea state 1 ($H_s = 15.7\,\text{m}$, $T_p = 17.8\,\text{s}$).



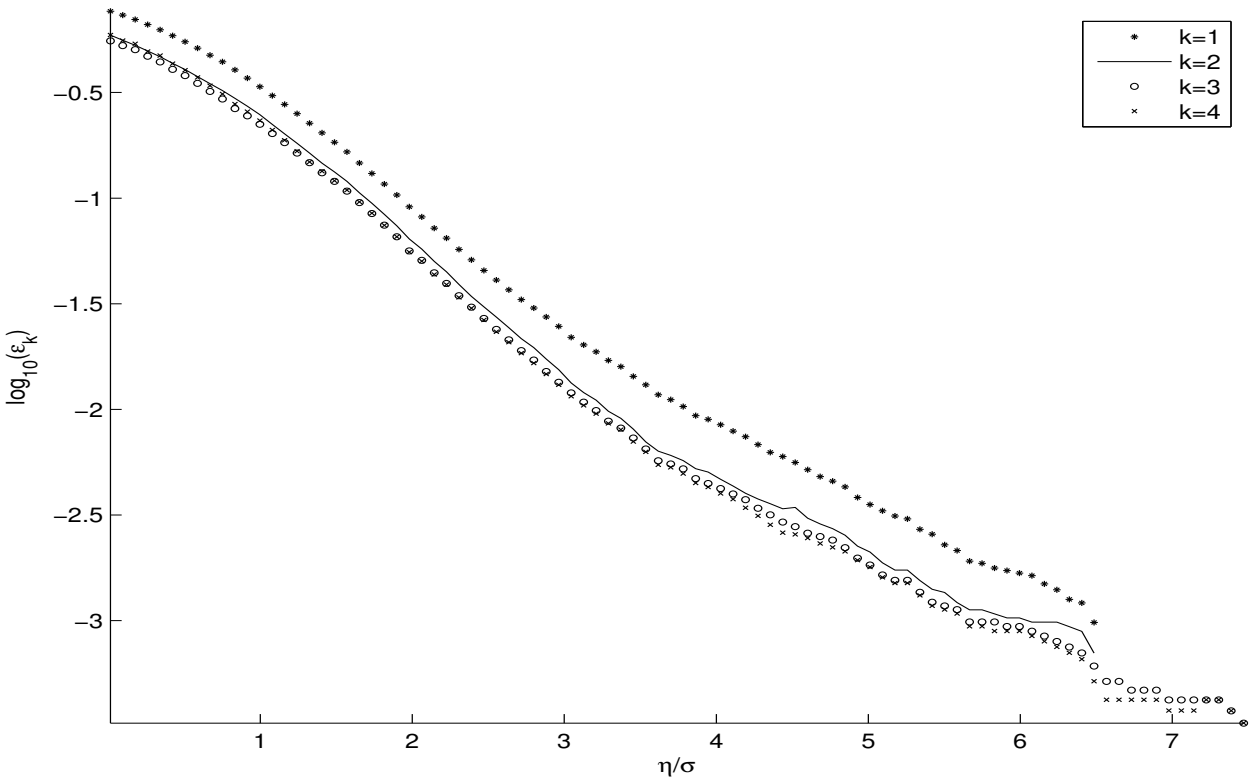
ACER

Log plot of ACER $\varepsilon_3(\eta)$ with extrapolation by optimally fitted curve, sea state 1. $c = 0.43$



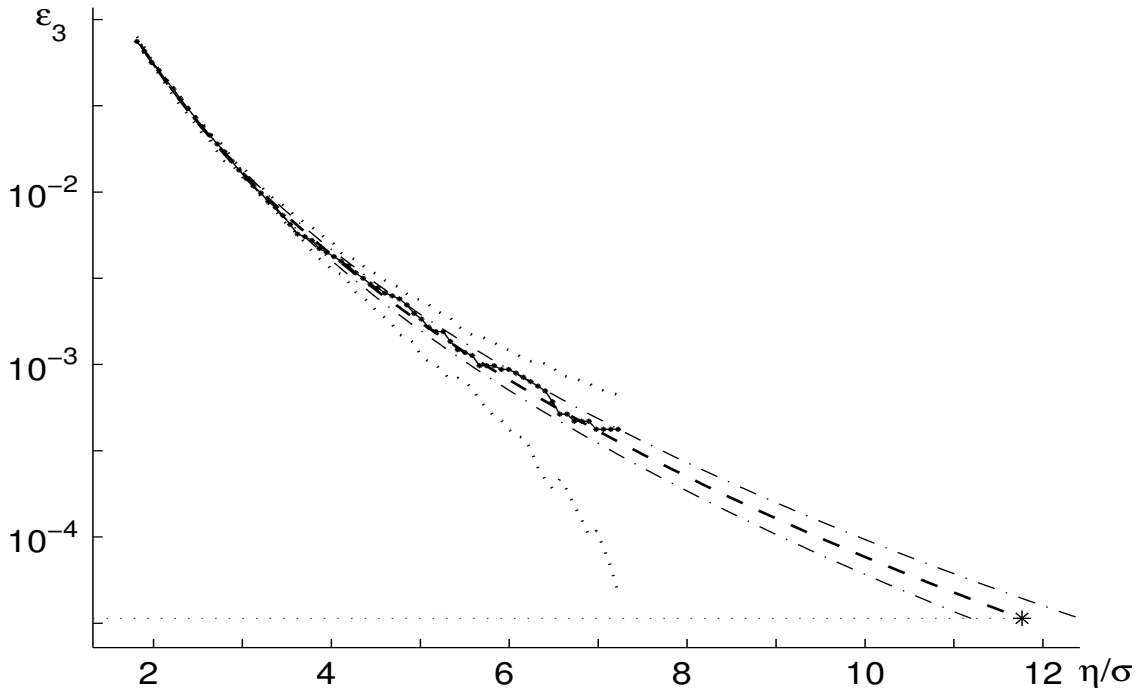
ACER

Log plot of ACER $\varepsilon_k(\eta)$, sea state 2 ($H_s = 15.0\,\text{m}$, $T_p = 16.7\,\text{s}$).



ACER

Log plot of ACER $\varepsilon_3(\eta)$ with extrapolation by optimally fitted curve, sea state 2. $c = 0.28$



Conclusions

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Conclusions

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- From the examples studied, it is tentatively concluded that the proposed extrapolation procedure combined with the ACER method appears to be accurate and robust, while it is simple and practical to use.
- Optimized fit and extrapolation can give accurate predictions of the ACER functions and thus extreme value statistics.