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Oppgave 5.10

X, Y uavh. eksp. ford. TV med parametre λ og μ.

$M = \min(X, Y)$

a) $\underline{E(MX \mid M = X)} = \int_0^\infty E(MX \mid M = X, X = x) f_X(x \mid M = X)\, dx$

$$= \int_0^\infty E(x^2) \frac{P(x < X \le x + dx,\ X \le Y)}{P(X \le Y)} = \int_0^\infty x^2 \frac{P(x < X \le x + dx,\ Y \ge x)}{P(X \le Y)}$$

$$= \int_0^\infty x^2 \frac{f_X(x)\, e^{-\mu x}\, dx}{\frac{\lambda}{\lambda + \mu}} = \int_0^\infty x^2 (\lambda + \mu) e^{-(\lambda+\mu)x} dx$$

$$= E(M^2) = \underline{\frac{2}{(\lambda + \mu)^2}}$$

b) $\underline{E(MX \mid M = Y)} = \int_0^\infty E(MX \mid M = Y, Y = y) f_Y(y \mid M = Y)\, dy$

$$= \int_0^\infty E(yX \mid X \ge y) \frac{P(y < Y \le y + dy,\ Y \le X)}{P(Y \le X)}$$

$$= \int_0^\infty y\, E(X \mid X \ge y) \frac{P(y < Y \le y + dy,\ X \ge y)}{P(Y \le X)}$$

$$= \int_0^\infty y (E(X) + y) \frac{f_Y(y)\, e^{-\lambda y}}{\frac{\mu}{\lambda + \mu}} dy = \int_0^\infty y\left(\frac{1}{\lambda} + y\right)(\lambda + \mu) e^{-(\lambda+\mu)y} dy$$

$$= \frac{1}{\lambda} E(M) + E(M^2) = \underline{\frac{1}{\lambda(\lambda + \mu)} + \frac{2}{(\lambda + \mu)^2}}$$

c)

$$\mathrm{Cov}(X,M) = E(XM) - E(X)E(M)$$

$$E(XM) = E(XM \mid M=X)P(M=X) + E(XM \mid M=Y)P(M=Y)$$

$$= \frac{2}{(\lambda+\mu)^2}\cdot\frac{\lambda}{(\lambda+\mu)} + \left(\frac{1}{\lambda(\lambda+\mu)} + \frac{2}{(\lambda+\mu)^2}\right)\frac{\mu}{\lambda+\mu}$$

$$= \frac{2}{(\lambda+\mu)^2} + \frac{\mu}{\lambda(\lambda+\mu)^2} = \frac{2\lambda+\mu}{\lambda(\lambda+\mu)^2}$$

$$\mathrm{Cov}(X,M) = \frac{2\lambda+\mu}{\lambda(\lambda+\mu)^2} - \frac{1}{\lambda}\cdot\frac{1}{\lambda+\mu} = \frac{\lambda}{\lambda(\lambda+\mu)^2} = \underline{\frac{1}{(\lambda+\mu)^2}}$$

<u>Korrelasj. koef.</u>

$$\rho_{XM} = \frac{\mathrm{Cov}(X,M)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(M)}} = \frac{\frac{1}{(\lambda+\mu)^2}}{\frac{1}{\lambda}\cdot\frac{1}{\lambda+\mu}} = \frac{\lambda}{\lambda+\mu} = P(X=M)$$

At $\rho_{XM} = P(X=M)$ kan synes rimelig.