FORSLAG TIL PROSJEKT-/HOVEDOPPGAVE
INDUSTRIELL MATEMATIKK

BIVARIATE EXTREME VALUE STATISTICS BASED ON THE ACER METHOD

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SUMMARY: The extension of extreme value statistics from the univariate to the bivariate case meets with several challenges. First of all, there is no direct generalization of the univariate extreme value types theorem to the bivariate case.

Developed from the Gumbel's logistic and mixed models, the later results on possible bivariate asymptotic extreme value distributions became in a sense too general. Many efforts have been made to model and estimate a function which describes the dependence structure between extreme components. However, there are no precise estimation tools that allow us to decide on the joint distribution of the bivariate extremes from a given set of bivariate data. Of course, the marginal data sets can be used to derive estimates of the marginal extreme value distributions, but the joint distribution is still a long way off.

A popular method of trying to cope with the problem of bivariate extremes is to adopt a copula to represent the joint distribution. For this purpose a range of different copulas have been proposed. Even in case of the bivariate extreme value copula, due to the properties of the dependence function, generally speaking there are an infinite number of models. Therefore, the main problem with this approach is that it is rather ad hoc. That is, there is no theoretical justification for choosing one particular copula over the other.

It is therefore of considerable interest to note that the concept of average conditional exceedance rate (ACER), which was developed during the last few years at NTNU, can be extended to several dimensions, in particular, to two. By this fact we obtain a vehicle for providing a nonparametric statistical estimate of the exact bivariate extreme value distribution given by a bivariate time series. It will be seen that the bivariate ACER is able to cover both spatial and temporal dependence characteristics of the given time series. Thus, it covers all simultaneous and non-simultaneous extreme events. From a practical point of view, this makes it possible to investigate the true behavior of the bivariate extreme value distribution for a particular case, and at the same time check the validity of the proposed copula models for bivariate extremes.

As the first effort in investigating the functional representation of the empirically estimated bivariate ACER surface, the bivariate extreme value copula approach will be investigated in the proposed project. Specifically, we shall start by looking at the Asymmetric logistic and Gumbel logistic models combined with the asymptotically consistent marginal extreme value distributions based on the univariate ACER functions.

The project may include some of the following elements:

1. Study the basis of multivariate extreme value statistics as it is commonly used.

2. Familiarize oneself with the multivariate ACER method for extreme value statistics.

3. Study the use of copulas for the representation of bivariate extreme value distributions.

4. Illustrate the results bu using wind speed data from Norwegian weather stations.